



A Conceptual Framework For Evaluating Capacity To Solve Algebraic Problems

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DOI - 10.5281/zenodo.8093371

Abstract:

The capacity to solve algebraic problems was brought up in conversation by a number of teachers and academics. Because it may be understood from a variety of vantage points, the capacity to solve algebraic problems does not have a single, definitive definition. In this article, we analyse the nature of algebraic problem-solving ability in terms of the algebraic processes that exhibit the capacity to solve algebraic problems. Specifically, we focus on the ability to solve linear algebra problems. On the basis of the three stages of algebraic processes, the historical evolution of algebra, and the SOLO model, a theoretical framework for the capacity to solve algebraic problems has been established (Structured of the Observed Learning Outcome). Investigating the pattern by gathering the numerical data was the first step in the three-step process of algebraic processes. This was followed by representing and generalising the pattern by creating a table and an equation, and the final step was interpreting and applying the equation to the related or new situation. Students' ability to solve algebraic equations is evaluated using a model called SOLO. This model has four levels of structure response: unistructural, multistructural, relational, and extended abstract. These levels incorporate two content domains of algebraic equations called direct variation and inverse variation.

Keywords: Algebraic Solving Ability, Algebraic Equation, SOLO Model, Direct Variation, Inverse Variation

Introduction:

The capacity to solve algebraic problems has been the topic of discussion among a large number of educators and scholars. Because it may be understood from a variety of vantage points, the capacity to solve algebraic problems does not have a single, universally accepted definition. In this article, we analyse the

nature of algebraic problem-solving ability in terms of the algebraic processes that exhibit the capacity to solve algebraic problems. Specifically, we focus on the ability to solve linear algebraic equations. On the basis of the three stages of algebraic processes, the historical evolution of algebra, and the SOLO model, a theoretical framework for the

capacity to solve algebraic problems was established (Structured of the Observed Learning Outcome). Investigating the pattern by gathering the numerical data was the first step in the three-step process of algebraic processes. This was followed by representing and generalising the pattern into a table and an equation, and the final step involved interpreting and applying the equation to a related or new circumstance. There are four levels (unistructural, multistructural, relational, and extended abstract) of structure response of the SOLO model that had been applied to assess students' ability to solve algebraic equations. These levels incorporate two content domains of algebraic equations, namely direct variation and inverse variation. The SOLO model had been used.

Literature Review:

During the first portion of the activity, students will be given a series of problems to solve that include specific scenarios. During their study with the numerical examples, it is anticipated of the students that they will observe and be able to identify any patterns that emerge (Friedlander & Hershkowitz, 1997). According to Lee (1996), the varying degrees of algebraic problem-solving abilities and viewpoints among students lead to a wide range of replies when they

are asked to work on a problem of this kind. Students then had the opportunity, during the second phase, to assess whether or not they would be able to express their numerical data into a table, which is a kind of algebraic representation that is often used (Friedlander & Hershkowitz, 1997; Herbert & Brown, 1997; Kaput, 1989). The representation offers a visual depiction of two variables that are linked together (the independent variable and the dependent variable), and it may assist individuals in recognising patterns (Herbert & Brown, 1997). In Swafford and Langrall's (2000) definition, tables are "systematic representations for a sequence of particular situations." When students are representing data in tables, it might offer them a feeling of the dynamic interaction that is taking place between the variables. Therefore, the degrees of students' knowledge of pattern may be set by the instructor depending on representation. This is something the teacher can do. After that, the students are tasked with utilising algebraic equations to symbolically generalise the connection that is present in the issue circumstance (Swafford & Langrall, 2000; Friedlander & Hershkowitz, 1997). To communicate generality in a challenging circumstance, one of the main senses to focus on is making generalisations via the examination of individual situations.

Students are likely perceiving the pattern via the specific number and the specific calculation, and they are aware of the generality of the situation. Friedlander and Hershkowitz (1997) and Mason (1996) observed that when students are presented with a "unfriendly" or large number of specific examples, it will push them to make a generalisation for the pattern, and they will be prompted to give the responses without having to see or draw them all. This is something that Friedlander and Hershkowitz (1997) and Mason (1996) found to be the case. Students had the opportunity to decide whether or not they would test the supposition, as well as the method by which they would do so, during the final portion of the activity. Students are needed to understand and use the equation in order to solve the related or new issue scenario in order to defend their findings throughout this step in the process. According to English and Warren (1995), the act of testing speculation helps improve the process of deductive reasoning. It establishes whether or not the students' premise or speculation led to a legitimate outcome. As a result, testing a conjecture paves the way for the meaningful use of algebraic manipulation as a component of higher-level algebraic problem-solving skills.

Methods:***Solo Model:***

The primary purpose of the SOLO model is to provide a means through which to evaluate the learning outcomes of pupils. The quality of learning is characterised using the SOLO taxonomy, which places an emphasis on the structure of an individual's response. SOLO is a framework that may be used to categorise the quality of a response based on the structure of the reaction to a stimulus. This inference can be made using SOLO. The SOLO model suggests that there are two aspects that should be considered while coding a student's answer. The first is a set of different ways that a person's cognitive abilities may grow, while the second is a set of different degrees of reaction.

Mode of Cognitive Development:

The concept of Piaget's stage of cognitive development, which proposes a number of developmental stages demonstrating increasing abstraction from sensori-motor (infancy), ikonic (early childhood of preschool), concrete-symbolic (childhood to adolescence), formal (early adulthood), through postformal (adulthood), is closely related to the mode in the SOLO model (Biggs & Collis, 1982; Biggs & Collis, 1989; Biggs & Teller, 1987; Collis & Romberg, 1986; Romberg, Zarinnia, & Collis, 1990).

Even if the progression of the five modes went from easy to difficult, it is a well-known fact that pupils do not always function at the same level as their developmental age implies they should and that they do not perform consistently (Biggs & Collis, 1982; Biggs & Teller, 1987; Collis & Romberg, 1986; Romberg, Zarinnia, & Collis, 1990). A student may, for instance, make a formal mode answer in Chemistry, which would then be followed by a sequence of concrete-symbolic mode responses in Biology. [Citation needed] In addition, a student's answer to a question in Economics that was posed using the concrete-symbolic method this month may be followed by a response using the formal style the following month. Was that specific pupil operating more in the concrete-symbolic mode or the formal style? According to the SOLO model, this sort of problem may be overcome by transferring the label from the student to his reaction to a specific task. This is one of the key components of the model (Biggs & Collis, 1982).

Discussion:

Research Related to The Using of Solo Model in Assessing Cognitive Development:

It has been argued, based on the research that has been done on the SOLO taxonomy, that SOLO is a hierarchical

model that is appropriate for assessing the learning result of a variety of various topics in the following way:

Science:

In a study that was carried out by Lake (1999), an adaptation of the SOLO model was described. This adaptation provides students and teachers with a pedagogically sound template that can be used to develop critical numerical skills, particularly interpretation of graphs and tables, in the context of the study of biology. In this context, the SOLO model is envisioned as a spiral learning structure that repeats itself with increasing degrees of abstraction. Each level builds on the knowledge and abilities that were learned at the level before it. Therefore, it was beneficial to be designed to classify the problem-solving processes by stages (unistructural, multistructural, relational, and extended abstract), and it was also beneficial to be adapted to provide a useful four-step template of generalised questions that led students from the basic skills to the critical analysis. Levins (1997) made an effort to demonstrate how the SOLO model could be used to appropriately classify the written responses of students into the cognitive classification framework for students of the same or different ages. These responses existed at varying degrees of consistency with regard to particular ideas regarding specific scientific

concepts. In this particular investigation, 190 students in grades 7 through 12 were asked three questions. The ability of the pupils to abstract became more developed from the first to the second cycle. This was reflected in the two cycles. In the first cycle, the students learned the fundamental ideas and abilities necessary for further study. Evaporation, for example, was described as having certain characteristics such as steam, water, heat, and gas. They also proposed some other characteristics. In other words, they responded in accordance with the reality that they saw. The pupils were able to comprehend the notions they had related their ideas of evaporation throughout the second cycle of the lesson. Therefore, the purpose of this research was to give an investigation into the development of the idea of evaporation within the conceptual parameters of the SOLO model. This approach included the development of a fundamental grasp of the concepts that are required to be in place prior to the shift to the more difficult abstract notions.

Practice Subject:

Chan, Tsui, and Chan made an effort to implement the SOLO paradigm into a practical situation (2002). The SOLO model was used to conduct an analysis on the scripts of lengthy essay papers and brief classroom discussion replies that were submitted by

postgraduate students who had studied an advanced practise topic in mental health. According to the findings, SOLO was appropriate for assessing the work in content variety of practise topics, and it was possible to be used to students who were at various stages of cognitive learning result. This was determined to be the case in the conclusion. They came to the conclusion that the categorization of levels should include sublevels, which should be included to the SOLO model. This would make the model less ambiguous and would boost agreement amongst raters (inter-rater reliability). To provide just a few examples, there is the prestructural, the unistructural, the multistructural moderate level, the multistructural high level, the relational moderate level, the relational high level, and the extended abstract level.

We assumed that the students are operating on the concrete-symbolic mode of the SOLO model since the research is focused on secondary school pupils. The transition from direct symbol systems of the world through oral language to the use of second order symbol systems such as written language signs and the mathematical symbol system is considered to be a significant shift in abstraction when it comes to the concrete-symbolic mode of learning. This mode of learning involves a more abstract process of learning and is

considered to be a significant shift in abstraction (Romberg & Collis, 1991; Wongyai & Kamol, 2004). Students may, for instance, become proficient with the idea of algebraic function. He is able to take 'x' to mean 'any number'. The student, on the other hand, has not had any prior experience with "any number," but rather has only had prior experience with "specific number" (Biggs & Collis, 1982). According to Biggs and Collis (1989), the most important goal in elementary and secondary education, regardless of the curriculum, is the mastering of symbol systems and the application of those systems to problems that occur in the actual world. Based on the SOLO model, we had a hypothesis that pupils in Form Four (Grade 10) might demonstrate four degrees of capacity to solve algebraic problems. These levels are called unistructural, multistructural, relational, and extended abstract. In addition to developing the theoretical framework, work had been done to determine the predicted degree of algebraic problem-solving abilities of students at each of the four levels for each of the academic areas.

Conclusion:

The framework that was established makes it possible for the students' abilities to solve algebraic problems to be expressed in a consistent

and methodical way. In spite of the large amount of study that has been done on the capacity of students to solve algebraic problems, contemporary research has not explored the two subject areas together. As a consequence of this, the existing body of research does not provide the type of consistent picture of students' capacity to solve algebraic problems that is required for the methodologies that are currently used in assessment and education.

Acknowledgement:

This investigation was made possible or assisted in part by JJTU funding. Even though impossible that some of our JJTU coworkers won agree with all of the interpretations or findings presented in this study, we would want to extend our gratitude to those of them who contributed valuable insights and areas of expertise to the research.

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