International Journal of Advance and Applied Research
www.ijaar.co.in
ISSN – 2347-7075 Impact Factor – 8.141
Peer Reviewed Bi-Monthly
Vol. 12 No.2 Nov-Dec 2024

Condensation of Hyperbolic Function & Curcular Function In Complex Domane

Mr. Manoj P Khere
Dr. Rajendra Gode Institute of Technology & Research, Amravati
Corresponding Author: Mr. Manoj P Khere
Email: manojkhere5@gmail.com
DOI- 10.5281/zenodo.14244229

Abstract:

This work is intended to introduce the problem of complex number and hyperbolic function generalized complex analysis. The algebraic properties of complex amylases are taken into consideration. Besides complex and hyperbolic generalized complex valued functions are defined and different equational representations of these numbers are examined. Moreover, the explanation of relationship between hyperbolic and circular function.

Keywords: Complex Number, Hyperbolic Function, Circular Function.

Introduction:

The solution of the Hyperbolic and Inverse Hyperbolic Function of the Complex number has been given in the S.V. Thakare book of Engineering Mathematics-1, In which they have used various trigonometric formulas and it is quite hard to learn. The same problem has been solved with less amount of trigonometrically and hyperbolic formulas, In a way which can be easily understood.

Complex numbers are defined as the set of all numbers z = x + yi, where x and y are real numbers. We denote the set of all complex numbers by z. We call x the real part of z. This is denoted by x = Re(z). We call y the imaginary part of z. This is denoted by y = Im(z). And complex analysis denote by w = f(z) = u + iv. We call u the real part of w. And we call v the imaginary part of w.

Brief Explanation:

The inverse hyperbolic function provides the hyperbolic angles corresponding to the given value of the hyperbolic function. This functions are denoted by $\sinh^{-1}$, $\cosh^{-1}$, $\tanh^{-1}$, csch^{-1}, sech^{-1}, and $\coth^{-1}$.

The formulae are explained for solving the below solution in easiest way:

Properties of Hyperbolic Functions:-

1. Sinh (-x) = -sinh x
2. Cosh (-x) = cosh x
3. Sinh 2x = 2 sinh x cosh x
4. Cosh 2x = $\cosh^2 x + \sinh^2 x$

The derivatives of hyperbolic functions are:

1. d/dx sinh (x) = cosh x   2. d/dx cosh (x) = sinh x

Some relations of hyperbolic function to the trigonometric function are as follows:

1. Sinh x = – i sin(ix)   2. Cosh x = cos (ix)   3. Tanh x = -i tan(ix)

Hyperbolic Function Identities

Pythagorean Trigonometric Identities

1. $\cosh^2(x) - \sinh^2(x) = 1$ 2. $\tanh^2(x) + \operatorname{sech}^2(x) = 1$ 3. $\coth^2(x) - \operatorname{cosech}^2(x) = 1$

Sum to Product

- sinh x + sinh y = 2 sinh((x+y)/2) cosh((x-y)/2)
- sinh x – sinh y = 2 cosh((x+y)/2) sinh((x-y)/2)
- cosh x + cosh y = 2 cosh((x+y)/2) cosh((x-y)/2)
- cosh x – cosh y = 2 sinh((x+y)/2) sinh((x-y)/2)

Product to Sum

- 2 sinh x cosh y = sinh(x + y) + sinh(x -y)
- 2 cosh x sinh y = sinh(x + y) – sinh(x – y)
- 2 sinh x sinh y = cosh(x + y) – cosh(x – y)
- 2 cosh x cosh y = cosh(x + y) + cosh(x – y).

Sum and Difference Identities

- sinh(x ± y) = sinh x cosh x ± coshx sinh y

- $\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
- $\tanh(x \pm y) = (\tanh x \pm \tanh y) / (1 \pm \tanh x \tanh y)$
- $\coth(x \pm y) = (\coth x \coth y \pm 1) / (\coth y \pm \coth x)$

Inverse Hyperbolic Functions

The inverse hyperbolic functions also called the area hyperbolic functions. It is also known as area hyperbolic function. The inverse hyperbolic function provides the hyperbolic angles corresponding to the given value of the hyperbolic function. This functions are denoted by $\sinh^{-1}$, $\cosh^{-1}$, $\tanh^{-1}$, csch^{-1}, sech^{-1}, and $\coth^{-1}$. The inverse hyperbolic function in complex number is defined as follows:

- $\text{Sinh}^{-1} x = \ln(x + \sqrt{[1+x^2]})$
- $\text{Cosh}^{-1} x = \ln(x + \sqrt{[x^2-1]})$
- $\text{Tanh}^{-1} x = (½)[\ln(1+x) – \ln(1-x)$

The Relationship between Hyperbolic and Circular Function:

The hyperbolic functions are corresponding of the trigonometric functions. The hyperbolic function use in the solutions of linear differential equations, calculation of distance and angles in the hyperbolic geometry, Laplace's equations in the Cartesian coordinates.

Problem Statemnet:

If $\tan\frac{x}{2} = \tanh\frac{u}{2}$ then show that $u = \log\left[\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right]$

Solution: - RHS=

$$= \log\left[\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right] \qquad \text{using } \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A * \tan B}$$

$$= \log\left[\frac{\tan\frac{x}{2} + \tan\frac{\pi}{4}}{1 - \tan\frac{x}{2} * \tan\frac{\pi}{4}}\right]$$

$$= \log\left[\frac{\tan\frac{x}{2} + 1}{1 - \tan\frac{x}{2}}\right]$$

$$= \log\left[\frac{1 + \tan\frac{x}{2}}{1 - \tan\frac{x}{2}}\right]$$

$$= \log\left[\frac{1 + \tanh\frac{u}{2}}{1 - \tanh\frac{u}{2}}\right] \qquad \text{by using given equation } \tan\frac{x}{2} = \tanh\frac{u}{2}$$

$$= \log\left[\frac{1 + \frac{e^{\frac{u}{2}} - e^{-\frac{u}{2}}}{e^{\frac{u}{2}} + e^{-\frac{u}{2}}}}{1 - \frac{e^{\frac{u}{2}} - e^{-\frac{u}{2}}}{e^{\frac{u}{2}} + e^{-\frac{u}{2}}}}\right]$$

$$= \log\left[\frac{e^{\frac{u}{2}} + e^{-\frac{u}{2}} + e^{\frac{u}{2}} - e^{-\frac{u}{2}}}{e^{\frac{u}{2}} + e^{-\frac{u}{2}} - (e^{\frac{u}{2}} - e^{-\frac{u}{2}})}\right]$$

$$= \log\left[\frac{e^{\frac{u}{2}} + e^{-\frac{u}{2}} + e^{\frac{u}{2}} - e^{-\frac{u}{2}}}{e^{\frac{u}{2}} + e^{-\frac{u}{2}} - e^{\frac{u}{2}} + e^{-\frac{u}{2}}}\right]$$

$$= \log\left[\frac{2e^{\frac{u}{2}}}{2\,e^{-\frac{u}{2}}}\right]$$

$$= \log\left[\frac{e^{\frac{u}{2}}}{e^{-\frac{u}{2}}}\right]$$

$$= \log e^{u}$$

$$= u$$

$$= \text{L.H.S}$$

Hence Prove

Mr. Manoj P Khere

Conclusion

This method explains the hyperbolic function in easiest way. I have used only one trigonometric formula in the whole problem statement. I have also used hyperbolic formula and logarithmic formulae to prove the solution. Complex analysis is an important part of the mathematical landscape because it connects many topics from the undergraduate curriculum. It can be used as a capstone course for mathematics majors as well as a stepping stone to independent research study of higher mathematics

Reference

1. Sobczyk, G., “The hyperbolic number plane”, The College Mathematics Journal, 26(4): 268-280, (1995).
2. Fjelstad, P., “Extending special relativity via the perplex numbers”, American Journal of Physics, 54(5): 416-422 (1986).
3. Study, E., Geometrie der Dynamen. Leipzig, (1903).
4. Pennestrì, E., Stefanelli, R., “Linear algebra and numerical algorithms using dual numbers”, Multibody System Dynamics, 18(3): 323-344 (2007).
5. Majernik, V., “Multicomponent number systems”, Acta Physica Polonica A, 90: 491-498, (1996).
6. Harkin, A. A., Harkin, J. B., “Geometry of generalized complex numbers”, Mathematics Magazine, 77(2): 118-129, (2004).
7. Cockle, J., “On a new imaginary in algebra”, Philosophical magazine, London-Dublin-Edinburgh, 34(226): 37-47, (1849).
8. Kantor, I., Solodovnikov, A., Hypercomplex Numbers, Springer-Verlag, New York, (1989).
9. Alfsmann, D., “On families of 2N -dimensional hypercomplex algebras suitable for digital signal processing”, 14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, (2006).
10. Conway, J. B. Functions of One Complex Variable, 2nd ed. New York: Springer-Verlag, 1995.
11. Forsyth, A. R. Theory of Functions of a Complex Variable, 3rd ed. Cambridge, England: Cambridge University Press, 1918.
12. Knopp, K. Theory of Functions Parts I and II, Two Volumes Bound as One, Part I. New York: Dover, 1996.
13. Krantz, S. G. Handbook of Complex Variables. Boston, MA: Birkhäuser, 1999.
14. Lang, S. Complex Analysis, 3rd ed. New York: Springer-Verlag, 1993.
15. Mathews, J. H. and Howell, R. W. Complex Analysis for Mathematics and Engineering, 5th ed. Sudbury, MA: Jones and Bartlett, 2006.
16. Variables." Ch. 4 in Methods of Theoretical Physics, Part I. New York: McGraw-Hill, pp. 348-491 and 480-485, 1953.
17. Weisstein, E. W. "Books about Complex Analysis.