International Journal of Advance and Applied Research (IJAAR)
Peer Reviewed Bi-Monthly
ISSN – 2347-7075
Impact Factor – 4.935
Vol.8 No.6 July – Aug 2021

MULTI-OBJECTIVE CAPACITATED TRANSPORTATION PROBLEM: A FUZZY APPROACH

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ABSTRACT:
The linear multi-objective capacitated transportation problem in which the supply and demand constraints are equality type, capacity restriction on each route are specified and the objectives are non- commensurable and conflict in nature. The fuzzy programing technique (Linear, Hyperbolic and Exponential) is used to find optimal compromise solution of a multi-objective capacitated transportation problem has been presented in this paper. An example is illustrate the methodology. Also comparison istaken out, using same example.
Keyword: Multi-criteria Decision Making, Capacitated Transportation Problem, Linear Membership Function, Non-linear Membership Function.

INTRODUCTION:

A transportation problem with capacity restriction is a linear programming problem. A basic solution to a capacitated transportation problem may contain more than m+n-2 positive values due to the capacity constraints which are additional to the m+n-2 independent equations. Fuzzy linear programming occurs when fuzzy set theory is applied to linear multi-criteria decision making problem. In fuzzy set theory, an element x has a degree of membership in a set A, denoted by a membership function (X). The range of the membership function is [0, 1]. Degree of the membership function for each objective represents its satisfaction level. If the membership function of an objective is one or zero then objective is fully achieved or not at all achieved,respectively. If the membership function of the objective lies in (0, 1) then the objective is partially achieved. In this paper, we present fuzzy programming with linear and hyperbolic membership function for solving multi-objective capacitated transportation problem.

MULTI-OBJECTIVE CAPACITATED TRANSPORTATION PROBLEM:

Consider m origins (i=1,2,…,m) and n destinations (j=1,2,…,n) at each origin O_i, let a_i be the amount of a homogeneous product which we want to transport to n destinations D_j to satisfy the demand for b_j units of the product there. A penalty c_{ij}^p is associated with transportation of a unit of the product from source i to destination j for the p-th criterion. The penalty could represent transportation cost, delivery time, quantity of goods delivered, under used capacity. A variable X_{ij} represents the unknown quantity to be transported from origin O_i to destination D_j. Let r_{ij} be the capacity restrictions on route i, j for capacitated transportation problem.

A multi-objective capacitated transportation problem can be represented as:

$$\text{Minimize } Z_p = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^p x_{ij} \qquad p=1,2,...,P \qquad (2.1)$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i=1,2,...,m \qquad (2.2)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j=1,2,...,n \qquad (2.3)$$

$$0 \le x_{ij} \le r_{ij} \qquad \text{for all } i,j \qquad (2.4)$$

Where the subscript on Z_p and superscript on c_{ij}^p denote p-th penalty criterion;

$a_i>0$ for all i $b_j>0$ for all j, $r_{ij} \ge 0$ for all i,j

And $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ as balanced condition.This balanced condition is necessary condition for the problem to have a feasible solution, however, this is not sufficient because of the condition (4.4).

For p=1, problem become to a single objective capacitated transportation problem. It may be considered as a special case of linear programming problem.

FUZZY PROGRAMMING TECHNIQUE FOR THE MULTI-OBJECTIVE CAPACITATED TRANSPORTATION PROBLEM:

Let U_p, L_p be the upper and lower bound for the p-th objective function, where lower bound indicates aspiration level of achievement and upper bound indicates highest acceptable level of achievement for the objective function respectively.

Let $d_p = (U_p - L_p)$ be degradation allowance for the Z_p objective.Once the aspiration levels and degradation allowance for each objective have been specified, we have formed the fuzzy model. Our next step is to transform the fuzzy model into a "Crisp" model.

Case i) Algorithm

Step 1:

Solve the multi-objective capacitated transportation problem as a single objective capacitated transportation problem using, each time, only one objective (ignore all others). Let $X^{1*} = \{x_{ij}^{1}\}$, $X^{2*} = \{x_{ij}^{2}\}$, ..., $X^{p*}=\{x_{ij}^{p}\}$ be the optimum solutions for pdifferent single objective capacitated transportation problem.

Step 2:

From the results of step 1, calculate the values of all the objective functions at all these $X^{1*}, X^{2*}, \ldots, X^{p*}$ optimal points. Then a payoff matrix is formed. The diagonal of the matrix constitutes individual optimum minimum values for the p objectives. The $X^{1*}, X^{2*}, \ldots, X^{p*}$ are the individual optimal solutions and each of these are used to determine the values of other individual objectives, thus the pay off matrix is developed as:

$$Z_1(X) \qquad Z_2(X) \quad \ldots \quad Z_p(X)$$

$$\begin{matrix} X^{(1)} \\ X^{(2)} \\ . \\ . \\ X^{(P)} \end{matrix} \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} \\ \cdots & \cdots & \cdots & \cdots \\ Z_{p1} & Z_{p2} & & Z_{pp} \end{bmatrix}$$

Step 3:

From step 2, we find for each objective, the lower bound (Lp) and upper bound (Up) corresponding to the sets of p solutions.

An initial fuzzy model of the problem (2.1-2.4) can be stated as: -

Find x_{ij}, i =1, 2, …,m; j = 1, 2, …,n;

So as to satisfy

$$Z_p \lesssim L_p \,, \qquad (3.1)$$

Subject to

$$\sum_{j=1}^{n} X_{ij} = a_i \,, \quad i = 1,2,\ldots,m \qquad (3.2)$$

$$\sum_{i=1}^{m} X_{ij} = b_j \,, \quad j = 1,2,\ldots,n \qquad (3.3)$$

$x_{ij} \geq 0$ for all i, j, k (3.4)

$\lesssim$ (fuzzification symbol) indicates nearly less than equal to

Step 4: Case (i)

Define a hyperbolic membership function $\mu_p^H(Z_p)$ for the p-th objective, are defined as follows

$$\mu_p^H(Z_p) = \frac{1}{2}\tanh\left(\left(\frac{U_p+L_p}{2} - Z_p\right)\alpha_p\right) + \frac{1}{2} \qquad (3.5)$$

where α_p is a parameter. Where $\alpha_p = \frac{3}{(U_p - L_p)/2} = \frac{6}{(U_p - L_p)}$

The hyperbolic membership function (3.5) has the following properties:

1. It is a strictly decreasing function.
2. It is a strictly concave for $Z_p \leq (U_p + L_p)/2$.
3. It is equal to 0.5 for $Z_p = (U_p + L_p)/2$.

4. It is a strictly convex for $Z_p \geq (U_p + L_p)/2,$

5. For all $X \in R^{mn}$ holds $o < \mu_p^H(Z_p) < 1,\ \mu_p^H(Z_p) = 1,$ is the lower asymptotic function of $\mu_p^H(Z_p),$; $\mu_p^H(Z_p)$ = 0, is the upper asymptotic function of $\mu_p^H(Z_p)$..

Step 5:

Formulate an equivalent nonlinear programming model with the help of the defined membership function (5.9) for the multi-objective capacitated transportation problem. This is stated as follows:

Maximize λ (3.6)

subject to

$\lambda \leq \mu_p^H(Z_p)$

$\sum_{j=1}^{n} X_{ij} = a_i$, i = 1,2,...,m (3.7)

$\sum_{i=1}^{m} X_{ij} = b_j$, j = 1,2,...,n (3.8)

$x_{ij} \geq 0$ for all i, j and $\lambda \geq 0$ (3.9)

where $\lambda = \underset{p}{\mathrm{Min}} \{\mu_p^H(Z_p)\}$

This is a nonlinear programming problem with one linear objective function, p non-linear and m+n+2mn+1 linear restriction. We shall now prove that there exists an equivalent linear programming problem.

Theorem: Define $X_{mn+1} = \tanh^{-1}(2\lambda - 1)$. The equivalent linear programming problem for the above nonlinear programming problem is as follows:

Maximize λ (3.10)

subject to

$\alpha_p Z_p + X_{mn+1} \leq \alpha_p (U_p + L_p)/2$ for all p. (3.11)

constraints (2.2),(2.3), (2.4) and $\lambda \geq 0$

Proof. For $t \in R$, we know $\tanh(t) = \dfrac{e^t - e^{-t}}{e^t + e^{-t}}$.

Therefore, nonlinear programming problem can be formulated as:

Maximize λ (3.12)

subject to

$$\lambda - \frac{1}{2}\tanh\left(\left(\frac{U_p+L_p}{2}-Z_p\right)\alpha_p\right) \leq \frac{1}{2} \quad (3.13)$$

constraints (2.2),(2.3), (2.4) and $\lambda \geq 0$

This is equivalent to

Maximize λ (3.14)

subject to

$$\tanh\left(\left(\frac{U_p+L_p}{2}-Z_p\right)\alpha_p\right) \geq 2\lambda-1 \quad (3.15)$$

Constraints **(**2.2),(4.3),(4.4)and $\lambda \geq 0$ (3.16)

Since tanh and $\tanh^{-1}$ are strictly increasing functions we have equivalently

Maximize λ (3.17)

subject to

$$\left(\frac{U_p+L_p}{2}-Z_p\right)\alpha_p \geq \tanh^{-1}(2\lambda-1) \quad (3.18)$$

constraints **(1.2), (1.3), (1.4) and** $\lambda \geq 0$ (3.19)

Or with $X_{mn+1}=\tanh^{-1}(2\lambda - 1)$

Maximize λ (3.20)

subject to

$$X_{mn+1} + \alpha_p Z_p \leq \left(\frac{U_p+L_p}{2}\right)\alpha_p \quad (3.21)$$

Constraints (1.2), (1.3), (1.4) and $\lambda \geq 0$ (3.22)

Because of $\lambda = \frac{\tanh(X_{mn+1})}{2} + \frac{1}{2}$ and the tanh function strictly increasing, it follows equivalently:

Maximize X_{mn+1} **(3.23)**

subject to

$$X_{mn+1} + \alpha_p Z_p \leq \left(\frac{U_p+L_p}{2}\right)\alpha_p \quad (3.24)$$

constraints (2.2),(2.3), (2.4) and $X_{mn+1} \geq 0$ (3.25)

This linear programming can be further simplified as:

Maximize X_{mn+1} (3.26)

subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{p} x_{ij} + \frac{X_{mn+1}}{\alpha_p} \leq \left(\frac{U_p + L_p}{2}\right) \quad (3.27)$$

constraints (2.2),(2.3), (2.4) and $X_{mn+1} \geq 0$

this model does not the form of a capacitated transportation problem.

The foregoing problem is a single objective linear programming that can be solved by linear programming algorithm

$\lambda^* = \frac{\tan x^*_{mn+1}}{2} + \frac{1}{2}$ and $Z(X^1) = \{z_1(x^1), z_2(x^1), \ldots z_p(x^1),\}$ is a non-dominated solution. This non dominated solution is an optimal compromise solution.

For integer optimal compromise solution we use model in step 5 with the mn integer.

Cases (ii) Linear Membership Function (iii) Exponential membership function are nearly same

Numerical Example:

$$\text{Minimize } Z_1 = 5X_{11} + 3X_{12} + 2X_{13} + 6X_{21} + 4X_{22} + 7X_{23} + 2X_{31} + 8X_{32} + 6X_{33}$$

$$\text{Minimize } Z_2 = 4X_{11} + 6X_{12} + 5X_{13} + 7X_{21} + 8X_{22} + 6X_{23} + 5X_{31} + 2X_{32} + 3X_{33}$$

$$\text{Minimize } Z_3 = 9X_{11} + 9X_{12} + 7X_{13} + 3X_{21} + 9X_{22} + 3X_{23} + 7X_{31} + 9X_{32} + 10X_{33}$$

$$\sum_{j=1}^{3} X_{1j} = 120 \quad ; \quad \sum_{j=1}^{3} X_{2j} = 145 \quad ; \quad \sum_{j=1}^{3} X_{3j} = 95$$

$$\sum_{i=1}^{3} X_{i1} = 80 \quad ; \quad \sum_{i=1}^{3} X_{i2} = 100 \quad ; \quad \sum_{i=1}^{3} X_{i1} = 180$$

$$X_{ij} \geq 0 \quad i=1,2,3. \ , \quad j=1,2,3.$$

Capacity restrictions of the routes are given as:

$$0 \leq x_{11} \leq 45, \ \ 0 \leq x_{12} \leq 60, \ \ 0 \leq x_{13} \leq 100$$
$$0 \leq x_{21} \leq 90, \ \ 0 \leq x_{22} \leq 100, 0 \leq x_{23} \leq 80$$
$$0 \leq x_{31} \leq 125, 0 \leq x_{32} \leq 85, \ \ 0 \leq x_{33} \leq 130$$

Step1 and step 2. Optimal solutions for minimizing the first objective Z_1

Subject to constraints () and () are as follows

$x_{11}= 20$, $x_{12}= 60$, $x_{13}= 40$, $x_{21}= 25$,
$x_{22} = 40$, $x_{23}= 80$, $x_{31}= 35$, $x_{33}= 60$
and other decision variable are zero
and $Z_1= 1660$

Optimal solutions for minimizing the second objective Z_2

Subject to constraints () and () are as follows

$x_{11}= 45$, $x_{12}= 35$, $x_{13}= 40$, $x_{21}= 35$,
$x_{22} = 30$, $x_{23}= 80$, $x_{32}= 35$, $x_{33}= 60$
and other decision variable are zero
and $Z_2= 1805$

Optimal solutions for minimizing the third objective Z_3

Subject to constraints () and () are as follows

$x_{11}= 20$, $x_{12}= 60$, $x_{13}= 40$, $x_{21}= 60$,
$x_{22} = 5$, $x_{23}= 80$, $x_{32}= 35$, $x_{33}= 60$
and other decision variable are zero
and $Z_3= 2380$

Now for $X^{(2)}$ we can find out Z_1, $Z_1(X^{(2)})=1935$

Now for $X^{(3)}$ we can find out Z_1, $Z_1(X^{(3)})=1940$

Now for $X^{(1)}$ we can find out Z_2, $Z_2(X^{(1)})=1570$

Now for $X^{(3)}$ we can find out Z_2, $Z_2(X^{(3)})=2190$

Now for $X^{(1)}$ we can find out Z_3, $Z_3(X^{(1)})=2670$

Now for $X^{(2)}$ we can find out Z_3, $Z_3(X^{(2)})=2530$

The payoff matrix is

$$\begin{array}{c} \\ X^{(1)} \\ X^{(2)} \\ X^{(3)} \end{array} \begin{array}{c} \begin{array}{ccc} Z_1 & Z_2 & Z_3 \end{array} \\ \begin{bmatrix} 1660 & 1570 & 2520 \\ 1935 & 1805 & 2530 \\ 1940 & 2190 & 2380 \end{bmatrix} \end{array}$$

U_1=1940, U_2=2190, U_3=2530
L_1=1660, L_2=1805, L_3=2380

Find $\{x_{ij},\ i=1,2,3;\ j=1,2,3\}$ so as satisfy
$Z_1 \lessapprox 1660$, $Z_2 \lessapprox 1805$, $Z_3 \lessapprox 2380$ and constraints (4.1),(4.2)

Step4. With $\alpha_p = \frac{6}{U_p - L_p}, \alpha_1 = \frac{6}{U_1 - L_1} = \frac{6}{280}, \alpha_2 = \frac{6}{U_2 - L_2} = \frac{6}{385}$

$$\alpha_3 = \frac{6}{U_3 - L_3} = \frac{6}{150}, \frac{U_1 + L_1}{2} = 1800,$$
$$\frac{U_2 + L_2}{2} = 1997.50, \frac{U_3 + L_3}{2} = 2455$$

We get the membership functions $\mu_1^H(Z_1), \mu_2^H(Z_2), \mu_3^H(Z_3)$ for the objectives Z_1,Z_2 and Z_3

Maximize X_{3x3+1}

Subject to

$$\alpha_1 Z_1(X) + X_{mn+1} \le \alpha_1\left(\frac{U_1 + L_1}{2}\right)$$

$$\frac{6}{280}(5X_{11}+3X_{12}+2X_{13}+6X_{21}+4X_{22}+7X_{23}+2X_{31}+8x_{32}+6X_{33})+X_{mn+1} \le \frac{6}{280}(1800)$$

$$30X_{11}+18X_{12}+12X_{13}+36X_{21}+24X_{22}+42X_{23}+12X_{31}+48x_{32}+36X_{33}+280X_{mn+1} \le 10800$$

Now,

$$\alpha_2 Z_2(X) + X_{mn+1} \le \alpha_2\left(\frac{U_2 + L_2}{2}\right)$$

$$\frac{6}{385}(4X_{11}+6X_{12}+5X_{13}+7X_{21}+8X_{22}+6X_{23}+5X_{31}+2x_{32}+3X_{33})+X_{mn+1} \le \frac{6}{385}(1997.5)$$

$$24X_{11}+36X_{12}+30X_{13}+42X_{21}+48X_{22}+36X_{23}+30X_{31}+12x_{32}+18X_{33}+385X_{mn+1} \leq 11985$$

And

$$\alpha_3 Z_3(X)+X_{mn+1} \leq \alpha_3(\frac{U_3+L_3}{2})$$

$$\frac{6}{150}(9X_{11}+9X_{12}+7X_{13}+3X_{21}+9X_{22}+3X_{23}+7X_{31}+9x_{32}+10X_{33})+X_{mn+1} \leq \frac{6}{150}(2455)$$

$$54X_{11}+54X_{12}+42X_{13}+18X_{21}+54X_{22}+18X_{23}+42X_{31}+54X_{32}+60X_{33}+150X_{mn+1} \leq 14730$$

The problem was solved by using the linear interactive and discrete optimization (LINDO) software, the optimal compromise solution is

$$X_{mn+1} = 0.1034$$

$$X^* = \begin{Bmatrix} x_{11}=20, x_{12}=60,\ x_{13}=40,\ x_{21}=41.896553,\ x_{22}=23.103449, \\ x_{23}=80,\ x_{31}=18.103449, x_{32}=16.896551\ x_{33}=60 \\ \end{Bmatrix}$$

$$Z_1^*=1789.3493\ ;\quad Z_2^*=1715.3103 \quad \text{and} \quad Z_3^*=2448.7931$$

$$\boxed{\lambda=0.55}$$

ii) Linear Membership Function

Find an equivalent crisp model

Maximize λ ,

$$Z_1(X)+280\lambda \leq 1940$$

$$5X_{11}+3X_{12}+2X_{13}+6X_{21}+4X_{22}+7X_{23}+2X_{31}+8x_{32}+6X_{33}+280\lambda \leq 1940$$

and

Maximize λ ,

$$Z_2(X)+385\lambda \leq 2190$$

$$4X_{11}+6X_{12}+5X_{13}+7X_{21}+8X_{22}+6X_{23}+5X_{31}+2x_{32}+3X_{33}+385\lambda \leq 2190$$

Maximize λ ,

$$9X_{11}+9X_{12}+7X_{13}+3X_{21}+9X_{22}+3X_{23}+7X_{31}+9x_{32}+10X_{33}+150\lambda \leq 2530$$

$$Z_3(X)+150\lambda \leq 2530$$

$$X^*=\left\{\begin{array}{l} x_{11}=20, x_{12}=60,\ x_{13}=40,\ x_{21}=41.896553,\ x_{22}=23.103449, \\ x_{23}=80,\ x_{31}=18.103449, x_{32}=16.896551\ x_{33}=60 \end{array}\right\}$$

$$Z_1^*=1789.3493\ ;\quad Z_2^*=1715.3103\quad \text{and}\quad Z_3^*=2448.7931$$

$$\boxed{\lambda=0.5172}$$

iii) Exponential Membership Function

Then an equivalent crisp model for fuzzy model can be formulated as

Maximize λ

subject to

$$\lambda\leq\frac{e^{-1\psi_p(x)}-e^{-1}}{1-e^{-1}}, \text{p = 1,2,-----P and subject to (7)-(9)}$$

$$\Psi_1(X)=\frac{Z_1-L_1}{U_1-L_1}=\frac{Z_1-1660}{1940-1660}=\frac{Z_1-1660}{280}$$

$$\Psi_2(X)=\frac{Z_2-L_2}{U_2-L_2}=\frac{Z_2-1805}{2190-1805}=\frac{Z_2-1805}{385}$$

$$\Psi_3(X)=\frac{Z_3-L_3}{U_3-L_3}=\frac{Z_3-2380}{2530-2380}=\frac{Z_3-2380}{150}$$

$$\Psi_1(X)=(5X_{11}+3X_{12}+2X_{13}+6X_{21}+4X_{22}+7X_{23}+2X_{31}+8x_{32}+6X_{33}-1660)/280$$

$$\Psi_2(X)=(4X_{11}+6X_{12}+5X_{13}+7X_{21}+8X_{22}+6X_{23}+5X_{31}+2x_{32}+3X_{33}-385)/385$$

$$\Psi_3(X)=(9X_{11}+9X_{12}+7X_{13}+3X_{21}+9X_{22}+3X_{23}+7X_{31}+9x_{32}+10X_{33}-2380)/150$$

Then the problem can be simplified as

$\Rightarrow$ Maximize λ

$$e^{-\Psi_1(X)}-(1-e^{-1})\lambda\geq e^{-1}\ \Rightarrow e^{-\Psi_1(X)}-(1-0.368)\lambda\geq 0.368\Rightarrow e^{-\Psi_1(X)}-(0.6321)\lambda\geq 0.368$$

$$e^{-\Psi_1(X)}-(1-e^{-1})\lambda\geq e^{-1}\ \Rightarrow e^{-\Psi_2(X)}-(1-0.368)\lambda\geq 0.368\Rightarrow e^{-\Psi_2(X)}-(0.6321)\lambda\geq 0.368$$

$$e^{-\Psi_1(X)}-(1-e^{-1})\lambda\geq e^{-1}\ \Rightarrow e^{-\Psi_3(X)}-(1-0.368)\lambda\geq 0.368\Rightarrow e^{-\Psi_3(X)}-(0.6321)\lambda\geq 0.368$$

The problem is solved by the (LINGO) software

$$X^* = \begin{Bmatrix} x_{12}=20, \; x_{13}=100, \; x_{21}=65, \; x_{23}=80, \; x_{31}=15, \; x_{32}=80. \\ \text{rest all } x_{ij} \text{ are zero's} \end{Bmatrix}$$

$Z_1^*=517.5$ and $Z_2^*=376.5$

$$\boxed{\lambda=0.8070}$$

And Ideal solution is {1660,1805,2380}. Also, set of non-dominated solutions {1660,1570,2520}; {1935,1805,2530}; {1940,2190,2380}

CONCLUSION:

We have obtained same optimal compromise solution by our proposed algorithm and fuzzy algorithm with membership functions (Bit et al. 1]) for the multi-objective capacitated transportation problem. For a multi-objective capacitated transportation problem with pobjective functions, the fuzzy programming with hyperbolic, linear and exponential membership function gives p non-dominated (efficient) solutions and an optimal compromise solution. The fuzzy programming algorithm with hyperbolic membership functions is applicable to multi-objective capacitated solid transportation problems and the vector minimum problems. This algorithm can be applied to the variants of multi-objective transportation problems similar linear multi objective programming problems. This paper is to be seen as a first step to introduce non-linear membership functions to a multi-objective capacitated transportation problem. The value of membership function of an objective represents the satisfaction level of the objective.

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