International Journal of Advance and Applied Research
www.ijaar.co.in
ISSN – 2347-7075 **Impact Factor – 8.141**
Peer Reviewed **Bi-Monthly**
Vol. 12 No.3 **Jan-Feb 2025**

The Mathematical Modeling of Performance Based Analysis of an Integrated Cellular and Ad Hoc Relay System

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DOI- 10.5281/zenodo.16759089

Abstract

In this paper, we propose a new wireless system architecture based on the integration of cellular and modern ad hoc relaying technologies. It can efficiently balance traffic loads and share channel resource between cells by using ad hoc relaying stations to relay traffic from one cell to another dynamically. However, the application demand and allocation could lead to congestion, if the network has to maintain such high resources for quality of service (QoS) requirements of the applications. In our system, handoff area and queue are taken into consideration and new and handoff calls are given priority, respectively. We analyze the system performance in terms of the call blocking probability and queueing delay for new call requests and call dropping probability for handoff requests. Numerical illustrations are provided with the help of Successive Overrelaxation Method (SOR). In order to improve the performance of base station, the trade off between number of services channel and QoS of base station must be considered.

Keywords: Ad Hoc networks, Cellular architecture, Relaying, Markovian model, Integration, Blocking probability, Queueing system modeling.

Introduction

Mobile communications have achieved rapid growth in recent years and the further advancement is expected to realize the future ubiquitous society. However, since the bandwidth is limited, it is very important to consider how to use the limited resources efficiently. Recently, the demand for wireless communications has grown tremendously and a lot of fundamental challenges and issues on wireless networks and mobile computing have been identified such as handover and call admission, fixed and dynamic channel assignment, data management, routing in wireless ad hoc networks, etc. As the demand of seamless communications is growing and the number of wireless users is increasing, effective call handling is becoming more and more important to utilize scarce radio resources more efficiently. In order to support dynamically arriving and departing calls effectively, size and hierarchy of cells, partitioned areas to handle wireless terminals, have to be carefully designed to maximize coverage area and to support user calls.

Ad hoc network systems are self organized, self managing, flexible and the multi hop communication in ad hoc networks leads to extending the coverage of existing wireless access technologies. It can be represented the improvement of reducing the cost of wireless access infrastructure. With the purpose of reusing the limited radio resources and reducing power consumption, a cellular networks require fixed base stations that are interconnected by a wired backbone and base stations are very important for the networks. Wu and Chuang (2001) studies dynamic QoS allocation for multimedia ad hoc wireless networks. Wu et al. (2003) analyzed a survey of mobile IP in cellular and mobile ad hoc network environments.[1] Remondo and Niemegeers (2003) gave ad hoc networking in future wireless communications. Zheng et al. (2004) performed recent advances in mobility modeling for mobile ad hoc networks. Ad hoc networks have multi-hop communication function and if the distance between two mobile stations is large, they can communicate with each other to relay stations. Murillo-Perez et al. (2009) studied the impact of mobility on OFDMA-based cellular systems with reuse partitioning. Sharma and Jain (2010) analyzed multihop cellular networks: a review. Pandey et al. (2013) studied optimum relay selection for energy efficient adhoc networks. Development of mobile adhoc network based on QOS routing protocol for healthcare was suggested by Hussain (2015). Guizani (2016) gave relay attacks concerns in wireless adhoc and sensor networks.

Cellular network system consists of two types of calls new calls and handover calls. The handoff calls are those calls, which are already ongoing and move into a new cell and need to connect to a new base station. The blocking probability of the hand off calls is an important demand direct, reliable and efficient connection. As the system, the study on the system with guard channels was represented by Guerin (1998). In this system, some number of channels is used exclusively for handoff calls because blocking of a call in progress is less desirable than the blocking of a new call. Zhang et al. (2003) have done approximation approach on performance evaluation for guard channel scheme. Yavuz and Leung (2006) presented computationally efficient method to evaluate the performance of guard-channel-based call admission control in cellular networks. Cruz-Perez et al. (2011) proposed approximated mathematical analysis methods of guard channel based call admission control in cellular networks. Kim et al. (2012) gave an analytical approach to the analysis of guard channel based call admission control in wireless cellular networks. Saritha, and Viswanatham (2014) presented a new approach for channel reservation and allocation to improve quality of service in vehicular communications.

Moreever, the channel reservation scheme that priority is given to handoff calls reduces the total carried traffic. The analysis of the system which some channels are reserved exclusively for handoff calls and there are the queues for the new and handoff calls. Jain (2000) presented prioritized channel assignment in mixed media cellular radio system. Salamah and Lababidi (2005) proposed dynamically adaptive channel reservation scheme for cellular network. Xhafa and Tonguz (2008) gave handover performance of priority schemes in cellular networks. Soh and Kim (2009) presented a predictive bandwidth reservation scheme using mobile positioning and road topology information. Sharma and Purohit (2011) suggested prioritized channel assignment to multimedia calls in wireless cellular networks.

Integrated cellular and ad hoc systems were represented in Wu et al. (2001). However, this paper focus on the coverage of a relay station and handoff area which is unstable, complicated and important is not taken into consideration. Fang and Chlamtac (2002) proposed analytical generalized results for handoff probability in wireless networks. Dharmraja et al. (2003) studied on modeling of wireless networks with general distributed handoff interarrival times. Wu et al. (2005) gave handoff performance of the integrated cellular and adhoc relaying (iCAR) system. Modeling of the system with handoff area represents the situation of a boundary in cells more than that of the system without handoff area. Bhattachary et al. (2008) gave traffic model and performance analysis of cellular mobile systems for general distributed handoff traffic and channel allocation. Kumar and Tripathi (2009) studied adaption of the preemptive handoff scheme in an integrated mobile communication environment. Halgamuge et al. (2011) performed handoff optimization using hidden markov model. Maximisation of correct handover probability and data throughput in vehicular networks was applied by Banda and Mzyece (2014).

It is not always possible to find analytic solution, as such numerical approaches thus appear to be the only way to obtain results. Garcia et al. (2005) studied admission control policies in the multi service cellular networks. Blocked handover sessions are queued up but an exponential deadline is defined beyond which a session is forced to terminate. We obtain on the numerical solution of the steady state Kolmogorov equations of the continuous time markov chain describing the system dynamics. Thus, the approximate model presented in this paper can be successfully used for an accurate performance evaluation, design and planning of cellular mobile telephone networks. In this paper, we consider the integrated ad hoc cellular network system where handoff area is taken into consideration and new and handoff calls are given priority. The rest of the paper is structured as follows. In section 2, Markovian model is described by stating the requisite assumptions and notations being used in the formulation of the mathematical model. The governing steady state equations are presented in section 4. Various performance measures are established in section 5. The steady state probability vectors are obtained by using successive overrelaxation method in the next section 6. Sensitivity analysis by taking numerical illustration is carried out by varying different parameters in section 7. Finally, we wind up our study by giving concluding remarks in section 8.

System Model

In this paper, there are three cells, cell A, cell B and cell C in this system. N channels are assigned to cell A, cell B and cell C, respectively. Handoff area is defined as the overlap region of cells and handoff calls are the calls that move to the neighbouring cell and handoff process is done within the handoff area. We consider the integrated ad hoc cellular network system, new calls are given the priority of relaying and new calls in handoff area and handoff calls are given the priority of queueing. When the relay station is set in handoff area and the area covered by the relay station is represented by the ratio l .

If there are no channels available in cell A, cell B and cell C on arrival, the new calls are covered by a relay station can be relayed to the other cell. New calls in handoff area and handoff calls use a channel for cell A or cell B or cell C with the probability of 0.3. New calls in handoff area and handoff calls can wait in a queue with capacity Q while they are in handoff area if there are no channels are available in cells A, B, C and they can not use the channels in the other cell. New calls in handoff area and handoff calls are blocked if the queue is full. If the calls in the queue can not get a new channel while they are in handoff area, they leave the queue halfway.

For the development of traffic model, we made the following assumptions:

1. There are N channels allocated in a each cell A, cell B and cell C, respectively
2. The arrival pattern of new calls and handoff calls follow the Poisson process.
3. The call holding times of new and handoff calls are exponential distributed.
4. The time between two successive handover requests of a call is a random variable and follows the exponential distribution.

We shall use the following notations for mathematical formulation purpose:

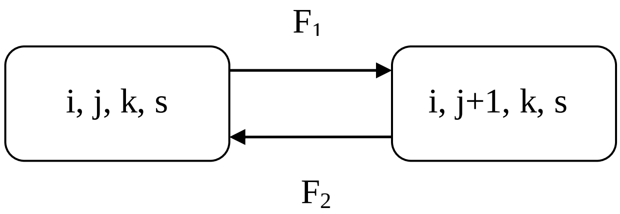
N	Total number of channels.
i	Number of being used channels of cell A
j	Number of being used channels of cell B
k	Number of being used channels of cell C
s	Number of calls in the queue
lnA	Arrival rate of new calls in cell A.
lnB	Arrival rate of new calls in cell B.
lnC	Arrival rate of new calls in cell C.
lhh	The arrival rate of handoff calls.
l¬nh	The arrival rate of handoff area.
l¬h	The arrival rate of new calls in handoff area and handoff calls.
m	Call holding times of new calls
md	Call holding times of handoff calls
mw	Dwell times of the cells and handoff area.
mtd	Service rate of releasing a channel.
mtw	Service rate of leaving a queue halfway.
Pi,j,k,s	The steady state probabilities of the cells.
Q	The capacity of queue.
l	The ratio of relay station of the cells with the probability 0.3.
BnA	Blocking probability of new calls in cell A.
BnB	Blocking probability of new calls in cell B.
BnC	Blocking probability of new calls in cell C.
Bh	Blocking probability of handoff calls.
Tch	Total carried traffic.

Define:	lh= lnh + lhh , mtd= m + md and mtw= m + mw

3. Queueing Network Model

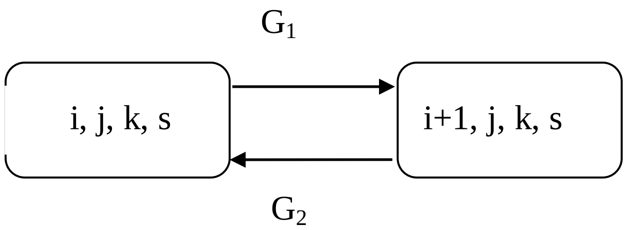
We define the state of the system as (i, j, k, s) where i denotes the number of being used channels of cell A, j does the number of being used channels of cell B, k does the number of being used channels of cell C and s does the number of call in the queue. M channels are assigned to cell A, cell B and cell C. The state dependent arrival rates and service rates for various states are shown in table 1, respectively.
The state dependent arrival rate and channel holding time is defined as

F:



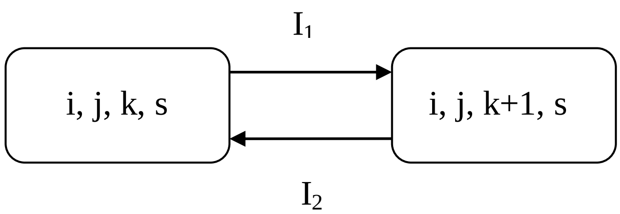
Condition	Arrival States	Service States
If i > j	$F_1=\square_{nB}+\square_h$	$F_2=(j+1)\,\square_{td}$
If i = j	$F_1=\square_{nB}+l\,\square_h$	$F_2=(j+1)\,\square_{td}$
If i < j	$F_1=\square_{nB}$	$F_2=(j+1)\,\square_{td}$
If i=M	$F_1=\square_{nB}+\square_h+l\,\square_{nA}$	$F_2=(j+1)\,\square_{td}$

G:



Condition	Arrival States	Service States
If j < i	$G_1=\square_{nA}$	$G_2=(i+1)\,\square_{td}$
If j = i	$G_1=\square_{nA}+l\,\square_h$	$G_2=(i+1)\,\square_{td}$
If j > i	$G_1=\square_{nA}+\square_h$	$G_2=(i+1)\,\square_{td}$
If j=M	$G_1=\square_{nA}+\square_h+l\,\square_{nB}$	$G_2=(i+1)\,\square_{td}$

I:



Condition	Arrival States	Service States
i, j > k, if $1 \le k \le M-1$,	I$_1$=□$_{nC}$ + □$_h$, i > k I$_1$=□$_{nC}$ j > k	I$_2$= (k+1) □$_{td}$
i = j=k, if $1 \le k \le M-1$,	I$_1$=□$_{nC}$ + l □$_h$	I$_2$= (k+1) □$_{td}$
i, j < k, if $1 \le k \le M-1$	I$_1$=□$_{nC}$, i < k I$_1$=□$_{nC}$ + □$_h$ j < k	I$_2$= (k+1) □$_{td}$
If i=k=M	I$_1$=□$_{nB}$ + □$_h$+l □$_{nA}$+ l □$_{nC}$	I$_2$= (j+1) □$_{td}$
If j=k=M	I$_1$=□$_{nA}$ + □$_h$+l □$_{nB}$+ l □$_{nC}$	I$_2$= (i+1) □$_{td}$
If i=j=M	I$_1$=□$_{nc}$ + □$_h$+l □$_{nA}$+ l □$_{nB}$	I$_2$= (k+1) □$_{td}$

$$\textbf{J:}\quad \begin{tikzcd} \boxed{M, M, M, s} \arrow[r, shift left, "J_1"] & \boxed{M, M,M,s+1} \arrow[l, shift left, "j_2"] \end{tikzcd}$$

□□□□□□□□□□□□□□□□□□□□□□□□□□□□ □□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□J$_1$□□□□□□$_h$	J$_2$= 2M□$_{td}$ +(s+1) □$_{tw}$

Table 1: Transition flow rates

4. Governing Equations

Using state transition rates as given in table 1 (A), for the computational purpose, the state governing equations for the fixed parameter M = 3 , Q=3 are constructed as given below:

$$-[(\lambda_{nA}+l\,\lambda_h)+(\lambda_{nB}+l\,\lambda_h)+(\lambda_{nC}+l\,\lambda_h)]P_{0,0,0,0}+\mu_{td}P_{1,0,0,0}+\mu_{td}P_{0,0,1,0}+\mu_{td}\,P_{0,1,0,0}=0 \qquad \ldots (1)$$

$$-[\lambda_{nB}+\lambda_{nC}+\mu_{td}+(\lambda_{nA}+\lambda_h)]P_{0,1,0,0}+\mu_{td}P_{0,1,1,0}+\mu_{td}P_{1,1,0,0}+2\mu_{td}\,P_{0,2,0,0}+(\lambda_{nB}+l\,\lambda_h)P_{0,0,0,0}=0 \qquad \ldots (2)$$

$$-[\lambda_{nB}+\lambda_{nC}+2\mu_{td}+(\lambda_{nA}+\lambda_h)]P_{0,2,0,0}+\mu_{td}P_{0,2,1,0}+\mu_{td}P_{1,2,0,0}+3\mu_{td}\,P_{0,3,0,0}+(\lambda_{nB})P_{0,1,0,0}=0 \qquad \ldots (3)$$

$$-[(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+\lambda_{nC})+(l\,\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+\lambda_{nC})+3\mu_{td}]P_{0,3,0,0}+\mu_{td}P_{1,3,0,0}$$
$$+\mu_{td}P_{0,3,1,0}+(\lambda_{nB})P_{0,2,0,0}=0 \qquad \ldots (4)$$

$$-[\lambda_{nB}+(\lambda_{nA}+l\,\lambda_h)+\mu_{td}+(\lambda_{nC}+\lambda_h)]P_{0,0,1,0}+\mu_{td}P_{0,1,1,0}+\mu_{td}P_{1,0,1,0}+2\mu_{td}\,P_{0,0,2,0}$$
$$+(\lambda_{nC}+l\,\lambda_h)P_{0,0,0,0}=0 \qquad \ldots (5)$$

$$-[(\lambda_{nA}+\lambda_h)+2\mu_{td}+(\lambda_{nB}+l\,\lambda_h)+\lambda_{nC}]P_{0,1,1,0}+\mu_{td}P_{1,1,1,0}+2\mu_{td}P_{0,1,2,0}+2\mu_{td}\,P_{0,2,1,0}$$
$$+(\lambda_{nC}+\lambda_h)P_{0,0,1,0}+\lambda_{nC}P_{0,1,0,0}=0 \qquad \ldots (6)$$

$$-\left[(\lambda_{nB})+2\mu_{td}+\mu_{td}+(\lambda_{nA}+l\,\lambda_h)+\lambda_{nC}\right]P_{0,1,2,0}+\mu_{td}P_{1,1,2,0}+2\mu_{td}P_{0,2,2,0}+3\mu_{td}\,P_{0,1,3,0} + (\lambda_{nB}+l\,\lambda_h)P_{0,0,2,0}+\lambda_{nC}P_{0,1,1,0}=0 \quad \ldots (7)$$

$$-\left[(\lambda_{nB})+3\mu_{td}+\mu_{td}+(\lambda_{nA}+l\,\lambda_h)\right]P_{0,1,3,0}+\mu_{td}P_{1,1,3,0}+2\mu_{td}P_{0,2,3,0}+(\lambda_{nB}+l\,\lambda_h)P_{0,0,3,0} + \lambda_{nC}P_{0,1,2,0}=0 \quad \ldots (8)$$

$$-\left[(\lambda_{n\mathbf{B}})+3\mu_{td}+(\lambda_{nA}+\lambda_h)+\lambda_{nC}\right]P_{0,2,1,0}+\mu_{td}P_{1,2,1,0}+2\mu_{td}P_{0,2,2,0}+3\mu_{td}\,P_{0,3,1,0} + (\lambda_{nB})P_{0,1,1,0}+\lambda_{nC}P_{0,2,0,0}=0 \quad \ldots (9)$$

$$-\left[(\lambda_{n\mathbf{B}}+l\,\lambda_h)+3\mu_{td}+(\lambda_{nA}+\lambda_h)+\lambda_{nC}\right]P_{0,2,2,0}+\mu_{td}P_{1,2,2,0}+3\mu_{td}P_{0,3,2,0}+3\mu_{td}\,P_{0,2,3,0} + (\lambda_{nB})P_{0,1,2,0}+\lambda_{nC}P_{0,2,1,0}=0 \quad \ldots (10)$$

$$-\left[(\lambda_{n\mathbf{B}}+\lambda_h)+2\mu_{td}+3\mu_{td}+(\lambda_{nA}+\lambda_h)\right]P_{0,2,3,0}+\mu_{td}P_{1,2,3,0}+3\mu_{td}P_{0,3,3,0} + (\lambda_{nB})P_{0,1,3,0}+(\lambda_{nC}+l\;\lambda_h)P_{0,2,2,0}=0 \quad \ldots (11)$$

$$-\left[(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC})+\mu_{td}+3\mu_{td}+(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_h+\lambda_{nC})\right]P_{0,3,1,0}+\mu_{td}P_{1,3,1,0} + 2\mu_{td}P_{0,3,2,0}+(\lambda_{nB})P_{0,2,1,0}+(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)P_{0,3,0,0}=0 \quad \ldots (12)$$

$$-\left[(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)+2\mu_{td}+3\mu_{td}+(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\lambda_{nC})\right]P_{0,3,2,0}+\mu_{td}P_{1,3,2,0} + 2\mu_{td}P_{0,3,2,0}+(\lambda_{nB}+l\,\lambda_h)P_{0,2,2,0}+(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)P_{0,3,1,0}=0 \quad \ldots (13)$$

$$-\left[(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC})+3\mu_{td}+3\mu_{td}\right]P_{0,3,3,0}+\mu_{td}P_{1,3,3,0} + (\lambda_{nB}+\lambda_h)P_{0,2,3,0}+(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)P_{0,3,2,0}=0 \quad \ldots (14)$$

$$-\left[\lambda_{nA}+(\lambda_{nC}+l\,\lambda_h)+\mu_{td}+(\lambda_{nB}+\lambda_h)\right]P_{1,0,0,0}+\mu_{td}P_{1,1,0,0}+\mu_{td}P_{1,0,1,0}+2\mu_{td}\,P_{2,0,0,0} + (\lambda_{nA}+l\,\lambda_h)P_{0,0,0,0}=0 \quad \ldots (15)$$

$$-\left[(\lambda_{nA}+l\,\lambda_h)+2\mu_{td}+(\lambda_{nB}+l\,\lambda_h)+\lambda_{nC}\right]P_{1,1,0,0}+\mu_{td}P_{1,1,1,0}+2\mu_{td}P_{1,2,0,0}+2\mu_{td}\,P_{2,1,0,0} + (\lambda_{nB}+\lambda_h)P_{1,0,0,0}+(\lambda_{nA}+\lambda_h)P_{0,1,0,0}=0 \quad \ldots (16)$$

$$-\left[(\lambda_{n\mathbf{B}})+2\mu_{td}+2\mu_{td}+(\lambda_{nA}+\lambda_h)+\lambda_{nC}\right]P_{1,2,0,0}+\mu_{td}P_{1,2,1,0}+2\mu_{td}P_{2,2,0,0}+3\mu_{td}\,P_{1,3,0,0} + (\lambda_{nB}+l\,\lambda_h)P_{1,1,0,0}+(\lambda_{nA}+\lambda_h)P_{0,2,0,0}=0 \quad \ldots (17)$$

$$-\left[(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC})+\mu_{td}+3\mu_{td}+(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)\right]P_{1,3,0,0}+2\mu_{td}P_{2,3,0,0} + \mu_{td}P_{1,3,1,0}+(\lambda_{nB})P_{1,2,0,0}+(\lambda_{nA}+l\,\lambda_{nB}+l\,\lambda_{nC}+\lambda_h)P_{0,3,0,0}=0 \quad \ldots (18)$$

$$-\left[(\lambda_{nB})+2\mu_{td}+(\lambda_{nC}+l\,\lambda_h)+\lambda_{nA}\right]P_{1,0,1,0}+\mu_{td}P_{1,1,1,0}+2\mu_{td}P_{2,0,1,0}+2\mu_{td}\,P_{1,0,2,0} + (\lambda_{nC}+l\;\lambda_h)P_{1,0,0,0}+\lambda_{nA}\,P_{0,0,1,0}=0 \quad \ldots (19)$$

$$-\left[(\lambda_{nB}+\lambda_h)+3\mu_{td}+(\lambda_{nC})+2\mu_{td}\right]P_{1,0,2,0}+\mu_{td}P_{1,1,2,0}+3\mu_{td}P_{1,0,3,0}+\lambda_{nA}\,P_{1,0,2,0} + (\lambda_{nA}+l\;\lambda_h)P_{0,0,2,0}+\lambda_{nc}\,P_{1,0,1,0}=0 \quad \ldots (20)$$

$$-\left[(\lambda_{nB}+\lambda_h)+3\mu_{td}+(\lambda_{nA})+\mu_{td}\right]P_{1,0,3,0}+\mu_{td}P_{1,1,3,0}+2\mu_{td}P_{2,0,3,0}+\lambda_{nC}\,P_{1,0,2,0} + (\lambda_{nA}+l\;\lambda_h)P_{0,0,3,0}=0 \quad \ldots (21)$$

$$-\left[(\lambda_{nB}+l\,\lambda_h)+\mu_{td}+(\lambda_{nA}+l\,\lambda_h)+\mu_{td}+(\lambda_{nC}+l\,\lambda_h)\right]P_{1,1,1,0}+2\mu_{td}P_{1,2,1,0}+2\mu_{td}P_{2,1,1,0}$$
$$+\lambda_{nC}P_{1,1,0,0}+(\lambda_{nA}+\lambda_h)P_{0,1,1,0}+\lambda_{nB}P_{1,0,1,,0}=0 \quad \ldots (22)$$

$$-\left[(\lambda_{nB})+\mu_{td}+(\lambda_{nA}+l\,\lambda_h)+\mu_{td}+(\lambda_{nC}+\lambda_h)\right]P_{1,1,2,0}+2\mu_{td}P_{1,2,2,0}+2\mu_{td}P_{2,1,2,0}$$
$$+(\lambda_{nB}+\lambda_h)P_{1,0,2,0}+(\lambda_{nA}+\lambda_h)P_{0,1,2,0}+(\lambda_{nC}+l\;\lambda_h)P_{1,1,1,,0}+3\mu_{td}P_{1,1,3,0}=0 \quad \ldots (23)$$

$$-\left[(\lambda_{nB})+\mu_{td}+(\lambda_{nA}+l\,\lambda_h)+\mu_{td}\right]P_{1,1,3,0}+2\mu_{td}P_{1,2,3,0}+2\mu_{td}P_{2,1,3,0}$$
$$+(\lambda_{nB}+\lambda_h)P_{1,0,3,0}+(\lambda_{nA}+\lambda_h)P_{0,1,3,0}=0 \quad \ldots (24)$$

$$-\left[(\lambda_{nB})+2\mu_{td}+(\lambda_{nA}+\lambda_h)+\mu_{td}+(\lambda_{nC})+\mu_{td}\right]P_{1,2,1,0}+3\mu_{td}P_{1,3,1,0}+2\mu_{td}P_{2,2,1,0}$$
$$+\lambda_{nC}P_{1,2,0,0}+(\lambda_{nB}+l\,\lambda_h)P_{1,1,1,0}+(\lambda_{nA}+\lambda_h)P_{0,2,1,,0}+2\mu_{td}P_{1,2,2,0}=0 \quad \ldots (25)$$

$$-\left[(\lambda_{nB})+2\mu_{td}+(\lambda_{nA}+\lambda_h)+\mu_{td}+(\lambda_{nC}+l\,\lambda_h)+\mu_{td}+2\mu_{td}\right]P_{1,2,2,0}+3\mu_{td}P_{1,2,3,0}+2\mu_{td}P_{2,2,2,0}$$
$$+\lambda_{nB}P_{1,1,2,0}+\lambda_{nC}P_{1,2,1,0}+(\lambda_{nB}+l\,\lambda_h)P_{1,1,1,0}+(\lambda_{nA}+\lambda_h)P_{0,2,2,,0}+2\mu_{td}P_{1,3,2,0}=0 \quad \ldots (26)$$

$$-\left[(\lambda_{nB})+2\mu_{td}+(\lambda_{nA}+\lambda_h)+\mu_{td}+3\mu_{td}\right]P_{1,2,3,0}+3\mu_{td}P_{1,3,3,0}+2\mu_{td}P_{2,2,3,0}$$
$$+\lambda_{nB}P_{1,1,3,0}+(\lambda_{nC}+l\,\lambda_h)P_{1,2,2,0}+(\lambda_{nA}+\lambda_h)P_{0,2,3,0}=0 \quad \ldots (27)$$

$$-\left[(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)+\mu_{td}+(\lambda_{nA}+l\,\lambda_{nB}+l\,\lambda_{nC}+\lambda_h)+3\mu_{td}+\mu_{td}\right]P_{1,3,1,0}+2\mu_{td}P_{1,3,2,0}$$
$$+2\mu_{td}P_{2,3,1,0}+\lambda_{nB}P_{1,2,1,0}+(\lambda_{nA}+l\,\lambda_{nB}+l\,\lambda_{nC}+\lambda)P_{0,3,1,0}+(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)P_{0,2,1,,0}=0 \quad \ldots (28)$$

$$-\left[(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)+\mu_{td}+(\lambda_{nA}+l\,\lambda_{nB}+l\,\lambda_{nC}+\lambda_h)+3\mu_{td}+2\mu_{td}\right]P_{1,3,2,0}+2\mu_{td}P_{2,3,2,0}$$
$$+3\mu_{td}P_{1,3,3,0}+(\lambda_{nB}+l\,\lambda_h)P_{1,2,2,0}+(\lambda_{nA}+l\,\lambda_{nB}+l\,\lambda_{nC}+\lambda)P_{0,3,2,0}+(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)P_{1,3,1,,0}=0 \quad \ldots (29)$$

$$-\left[\mu_{td}+(\lambda_{nA}+l\,\lambda_{nB}+l\,\lambda_{nC}+\lambda_h)+3\mu_{td}+3\mu_{td}\right]P_{1,3,3,0}+2\mu_{td}P_{2,3,3,0}$$
$$+(\lambda_{nB})P_{1,2,3,0}+(\lambda_{nA}+l\,\lambda_{nB}+l\,\lambda_{nC}+\lambda)P_{0,3,3,0}+(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)P_{1,3,2,,0}=0 \quad \ldots (30)$$

$$-\left[(\lambda_{nB}+\lambda_h)+2\mu_{td}+(\lambda_{nA})+(\lambda_{nC}+l\,\lambda_h)\right]P_{2,0,0,0}+\mu_{td}P_{2,0,1,0}+\mu_{td}P_{2,,1,0,0}$$
$$+\lambda_{nA}P_{1,0,0,0}+3\mu_{td}P_{3,0,0,0}=0 \quad \ldots (31)$$

$$-\left[(\lambda_{nB}+\lambda_h)+2\mu_{td}+\mu_{td}+(\lambda_{nA})+(\lambda_{nC})\right]P_{2,1,0,0}+2\mu_{td}P_{2,2,0,0}+\mu_{td}P_{2,1,,1,0}$$
$$+(\lambda_{nB}+\lambda_h)P_{2,0,0,0}+(\lambda_{nA}+l\,\lambda_h)P_{1,1,0,0}+3\mu_{td}P_{3,1,0,0}=0 \quad \ldots (32)$$

$$-\left[(\lambda_{nB}+l\,\lambda_h)+2\mu_{td}+2\mu_{td}+(\lambda_{nA}+l\,\lambda_h)+(\lambda_{nC})\right]P_{2,2,0,0}+3\mu_{td}P_{2,3,0,0}+\mu_{td}P_{2,2,,1,0}$$
$$+(\lambda_{nB}+\lambda_h)P_{2,1,0,0}+(\lambda_{nA}+\lambda_h)P_{1,2,0,0}+3\mu_{td}P_{3,2,0,0}=0 \quad \ldots (33)$$

$$-\left[(\lambda_{nA}+l\,\lambda_{nB}+l\lambda_{nC}+\lambda_h)+2\mu_{td}+3\mu_{td}+(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)\right]P_{2,3,0,0}+3\mu_{td}P_{3,3,0,0}+\mu_{td}P_{2,3,,1,0}$$
$$+(\lambda_{nA}+l\,\lambda_{nB}+l\lambda_{nC}+\lambda_h)P_{1,3,0,0}+(\lambda_{nB})P_{2,2,0,0}=0$$

… (34)

$$-\left[\left(\lambda_{nC}+\lambda_h\right)+\mu_{td}+2\mu_{td}+\left(\lambda_{nA}\right)+\left(\lambda_{nB}+\lambda_h\right)\right]P_{2,0,1,0}+2\mu_{td}P_{2,0,2,0}+\mu_{td}P_{2,1,,1,0}$$
$$+\left(\lambda_{nC}+l\,\lambda_h\right)P_{2,0,0,0}+\left(\lambda_{nA}\right)P_{1,0,1,0}+3\mu_{td}P_{3,0,1,0}=0 \quad \ldots (35)$$

$$-\left[\left(\lambda_{nB}+\lambda_h\right)+2\mu_{td}+2\mu_{td}+\left(\lambda_{nA}\right)+\left(\lambda_{nC}\right)\right]P_{2,0,2,0}+3\mu_{td}P_{2,0,3,0}+\mu_{td}P_{2,1,2,0}$$
$$+\left(\lambda_{nA}\right)P_{1,0,2,0}+\left(\lambda_{nC}\right)P_{2,0,1,0}+3\mu_{td}P_{3,0,2,0}=0 \quad \ldots (36)$$

$$-\left[\left(\lambda_{nB}+\lambda_h\right)+2\mu_{td}+3\mu_{td}+\left(\lambda_{nA}\right)\right]P_{2,0,3,0}+3\mu_{td}P_{3,0,3,0}+\mu_{td}P_{2,1,3,0}$$
$$+\left(\lambda_{nA}\right)P_{1,0,3,0}+\left(\lambda_{nC}\right)P_{2,0,2,0}=0 \quad \ldots (37)$$

$$-\left[\left(\lambda_{nB}+\lambda_h\right)+\mu_{td}+2\mu_{td}+\left(\lambda_{nA}\right)+\left(\lambda_{nC}+l\,\lambda_h\right)+\mu_{td}\right]P_{2,1,1,0}+2\mu_{td}P_{2,2,1,0}+3\mu_{td}P_{3,1,,1,0}$$
$$+\left(\lambda_{nB}+\lambda_h\right)P_{2,0,1,0}+\left(\lambda_{nA}+l\,\lambda_h\right)P_{1,1,1,0}+2\mu_{td}P_{2,1,2,0}+\lambda_{nC2,1,0,0}=0 \quad \ldots (38)$$

$$-\left[\left(\lambda_{nC}+l\,\lambda_h\right)+\mu_{td}+2\mu_{td}+\left(\lambda_{nA}\right)+\left(\lambda_{nB}\right)+2\mu_{td}\right]P_{2,1,2,0}+3\mu_{td}P_{3,1,2,0}+2\mu_{td}P_{2,1,2,0}$$
$$+\left(\lambda_{nA}+l\,\lambda_h\right)P_{2,1,2,0}+\left(\lambda_{nB}+\lambda_h\right)P_{2,0,2,0}+3\mu_{td}P_{2,1,3,0}+\lambda_{nC}P_{2,1,1,0}=0 \quad \ldots (39)$$

$$-\left[\left(\lambda_{nA}\right)+\mu_{td}+2\mu_{td}+\left(\lambda_{nB}\right)+3\mu_{td}\right]P_{2,1,3,0}+3\mu_{td}P_{3,1,3,0}+2\mu_{td}P_{2,2,3,0}$$
$$+\left(\lambda_{nA}+l\,\lambda_h\right)P_{1,1,3,0}+\left(\lambda_{nB}+\lambda_h\right)P_{2,0,3,0}+\left(\lambda_{nC}+l\,\lambda_h\right)P_{2,1,2,0}=0 \quad \ldots (40)$$

$$-\left[\left(\lambda_{nB}+l\,\lambda_h\right)+2\mu_{td}+2\mu_{td}+\left(\lambda_{nA}+l\,\lambda_h\right)+\left(\lambda_{nC}\right)+\mu_{td}\right]P_{2,2,1,0}+3\mu_{td}P_{2,3,1,0}+3\mu_{td}P_{3,2,,1,0}$$
$$+\left(\lambda_{nB}+\lambda_h\right)P_{2,1,1,0}+\left(\lambda_{nA}+\lambda_h\right)P_{1,2,1,0}+2\mu_{td}P_{2,2,2,0}+\lambda_{nC}P_{2,2,0,0}=0 \quad \ldots (41)$$

$$-\left[\left(\lambda_{nB}+l\,\lambda_h\right)+2\mu_{td}+2\mu_{td}+\left(\lambda_{nA}+l\,\lambda_h\right)+\left(\lambda_{nC}+l\,\lambda_h\right)+2\mu_{td}\right]P_{2,2,2,0}+3\mu_{td}P_{2,2,3,0}+3\mu_{td}P_{3,2,2,0}$$
$$+\left(\lambda_{nA}+\lambda_h\right)P_{1,2,2,0}+\left(\lambda_{nC}+\lambda_h\right)P_{2,2,1,0}+2\mu_{td}P_{2,1,2,0}+3\mu_{td}P_{2,3,2,0}=0$$

… (42)

$$-\left[\left(\lambda_{nB}+l\,\lambda_h\right)+2\mu_{td}+2\mu_{td}+\left(\lambda_{nA}+l\,\lambda_h\right)+3\mu_{td}\right]P_{2,2,3,0}+3\mu_{td}P_{3,2,3,0}+3\mu_{td}P_{2,3,3,0}$$
$$+\left(\lambda_{nA}+\lambda_h\right)P_{1,2,3,0}+\left(\lambda_{nC}+l\,\lambda_h\right)P_{2,2,2,0}+3\mu_{td}P_{2,3,2,0}=0 \quad \ldots (43)$$

$$-\left[\left(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h\right)+\mu_{td}+2\mu_{td}+\left(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC}\right)+3\mu_{td}\right]P_{2,3,1,0}+2\mu_{td}P_{2,3,2,0}$$
$$+\left(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC}\right)P_{1,3,1,0}+\left(\lambda_{nB}\right)P_{2,2,1,0}+3\mu_{td}P_{3,3,1,0}+\left(l\,\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+\lambda_{nC}\right)P_{2,3,0,0}=0$$

… (44)

$$-\left[\left(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h\right)+2\mu_{td}+2\mu_{td}+\left(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC}\right)+3\mu_{td}\right]P_{2,3,2,0}+3\mu_{td}P_{3,3,2,0}$$
$$+\left(l\,\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+\lambda_{nC}\right)P_{2,3,1,0}+\left(\lambda_{nB}+\lambda_h\right)P_{2,2,2,0}+3\mu_{td}P_{2,3,3,0}+\left(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC}\right)P_{1,3,2,0}=0$$

… (45)

$$-\left[\left(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h\right)+3\mu_{td}+\left(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC}\right)+\left(\lambda_{nB}+\lambda_h\right)\right]P_{2,3,3,0}+3\mu_{td}P_{3,3,2,0}$$
$$+\left(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC}\right)P_{1,3,3,0}+\left(\lambda_{nB}+\lambda_h\right)P_{2,2,3,0}+3\mu_{td}P_{3,3,3,0}+\left(l\,\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+\lambda_{nC}\right)P_{2,3,2,0}=0$$

… (46)

$$-\left[\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nC}+l\,\lambda_{nA}\right)+3\mu_{td}+\left(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC}\right)\right]P_{3,0,0,0}+\mu_{td}P_{3,1,0,0}$$
$$+\left(\lambda_{nA}\right)P_{2,0,0,0}+\mu_{td}P_{3,0,1,0}=0 \quad \ldots (47)$$

$$-\left[\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)+3\mu_{td}+\mu_{td}+\mu_{td}\right]P_{3,0,1,0}+\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)P_{3,0,0,0}$$
$$+2\mu_{td}P_{3,0,2,0}+\lambda_{nA}P_{2,0,1}+\mu_{td}P_{3,1,1,0}=0 \quad \ldots (48)$$

$$-\left[\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)+3\mu_{td}+2\mu_{td}+\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)\right]P_{3,0,2,0}+3\mu_{td}P_{3,0,3,0}$$
$$+\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)P_{3,0,1,0}+\mu_{td}P_{3,1,2,0}+\lambda_{nA}P_{2,0,2,0}=0 \quad \ldots (49)$$

$$-\left[\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)+3\mu_{td}+3\mu_{td}\right]P_{3,0,3,0}+\mu_{td}P_{3,1,3,0}+\lambda_{nA}P_{2,0,3}$$
$$+\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)P_{3,0,2,0}=0 \quad \ldots (50)$$

$$-\left[\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)+3\mu_{td}+\mu_{td}\right]P_{3,1,0,0}+2\mu_{td}P_{3,2,0,0}+\lambda_{nA}P_{2,1,0}$$
$$+\left(\lambda_{nb}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)P_{3,1,0,0}=0 \quad \ldots (51)$$

$$-\left[\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)+3\mu_{td}+2\mu_{td}\right]P_{3,2,0,0}+3\mu_{td}P_{3,3,0,0}+\left(\lambda_{nA}+l\,\lambda_h\right)P_{2,2,0}$$
$$+\left(\lambda_{nb}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)P_{3,1,0,0}=0 \quad \ldots (52)$$

$$-\left[\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)+3\mu_{td}+3\mu_{td}\right]P_{3,3,0,0}+\mu_{td}P_{3,3,1,0}+\left(\lambda_{nA}+\lambda_h\right)P_{2,3,0}$$
$$+\left(\lambda_{nb}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)P_{3,2,0,0}=0 \quad \ldots (53)$$

$$-\left[\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)+3\mu_{td}+\mu_{td}+\mu_{td}+\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)\right]P_{3,1,1,0}+2\mu_{td}P_{3,1,2,0}$$
$$+\left(\lambda_{nA}\right)P_{2,1,1}+\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)P_{3,0,1,0}+2\mu_{td}P_{3,2,1,0}=0 \quad \ldots (54)$$

$$-\left[\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)+3\mu_{td}+\mu_{td}+2\mu_{td}+\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)\right]P_{3,1,2,0}+2\mu_{td}P_{3,2,2,0}$$
$$+\left(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC}\right)P_{2,1,2}+\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)P_{3,0,2,0}+\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)P_{3,1,2,0}$$
$$+3\mu_{td}P_{3,1,3,0}=0 \quad \ldots (55)$$

$$-\left[\mu_{td}+3\mu_{td}+3\mu_{td}+\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)\right]P_{3,1,3,0}+2\mu_{td}P_{3,2,3,0}$$
$$+\left(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC}\right)P_{2,1,3}+\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)P_{3,0,3,0}+2\mu_{td}P_{3,2,3,0} \quad \ldots (56)$$
$$+\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)P_{3,1,2,0}=0$$

$$-\left[\mu_{td}+3\mu_{td}+2\mu_{td}+\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)+3\mu_{td}\right]P_{3,2,1,0}+2\mu_{td}P_{3,2,2,0}$$
$$+\left(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC}\right)P_{2,2,1}+\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)P_{3,1,1,0}+\left(\lambda_{nB}+\lambda_h\right)P_{3,2,1,0} \quad \ldots (57)$$
$$+\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)P_{3,2,0,0}=0$$

$$-\left[2\mu_{td}+3\mu_{td}+2\mu_{td}+\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)+\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)\right]P_{3,2,2,0}+3\mu_{td}P_{3,3,2,0}$$
$$+\left(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC}\right)P_{2,2,2}+\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)P_{3,1,2,0}+3\mu_{td}P_{3,2,3,0}$$
$$+\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)P_{3,2,1,0}=0 \quad \ldots (58)$$

$$-\left[3\mu_{td}+3\mu_{td}+2\mu_{td}+\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)\right]P_{3,2,3,0}+3\mu_{td}P_{3,3,3,0}$$
$$+\left(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC}\right)P_{2,2,3}+\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)P_{3,1,3,0}+\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)P_{3,2,2,0}=0 \quad \ldots (59)$$

$$-\left[3\mu_{td}+3\mu_{td}+\mu_{td}+\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)\right]P_{3,3,1,0}+2\mu_{td}P_{3,3,2,0}$$
$$+\left(\lambda_{nA}+\lambda_h\right)P_{2,3,1,0}+\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)P_{3,2,1,0}+\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)P_{3,3,0,0}=0 \quad \ldots (60)$$

$$-\left[3\mu_{td}+3\mu_{td}+2\mu_{td}+\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)\right]P_{3,3,2,0}+3\mu_{td}P_{3,3,3,0}+\left(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC}\right)$$
$$P_{2,3,2,0}+\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)P_{3,2,2,0}+\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)P_{3,3,1,0}=0 \quad \ldots (61)$$

$$-\left[3\mu_{td}+3\mu_{td}+3\mu_{td}\right]P_{3,3,3,0}+\left(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC}\right)P_{2,3,3,0}+\left(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC}\right)P_{3,2,3,0}+\left(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB}\right)P_{3,3,2,0}=0 \quad \ldots(62)$$

$$-\left[\left(3M\,\mu_{td}+(r+1)\mu_{tw}\right)\right]P_{3,3,3,r}+\left(\lambda_h\right)P_{3,3,3,r-1}=0 \qquad 1\le r\le Q \quad \ldots(63)$$

5. Performance Measures

Many interesting performance indices can be computed using the steady state probabilities. The most important performance measure for our systems is the probability that a call is blocked due to lack of available resources. Using probabilities obtained from equations given in previous section, we can establish various performance measures as follows

- *The blocking probability of the new calls in cell A is obtained by*
- $$B_{nA}=\sum_{s=0}^{Q}P_{M,M,M,s}+\sum_{j=0}^{M-1}P_{M,j,M,0}(1-l)+\sum_{k=0}^{M-1}P_{M,M,k,0}(1-l) \quad \ldots(64)$$
- *The blocking probability of the new calls in cell B is obtained by*
- $$B_{nB}=\sum_{s=0}^{Q}P_{M,M,M,s}+\sum_{i=0}^{M-1}P_{i,M,M,0}(1-l)+\sum_{k=0}^{M-1}P_{M,M,k,0}(1-l) \quad \ldots(65)$$
- *The blocking probability of the new calls in cell C is obtained by*
- $$B_{nC}=\sum_{s=0}^{Q}P_{M,M,M,s}+\sum_{i=0}^{M-1}P_{i,M,M,0}(1-l)+\sum_{j=0}^{M-1}P_{M,j,M,0}(1-l) \quad \ldots(66)$$

- *The blocking probability of the new calls in handoff area and handoff calls*
- *is obtained by*
- $B_h = P_{M,M,M,Q}$ 67
- *The total carried load T_{ch} in cell A, cell B and cell C is obtained by*
- $$T_{ch}=\sum_{i=0}^{M}\sum_{j=0}^{M}\sum_{k=0}^{M}(i+j+k)P_{i,j,k,0}+\sum_{s=1}^{Q}2MP_{M,M,M,s}$$ 68

6. Successive Overrelaxation Method for a Linear System

Successive overrelaxation method is an iterative technique that uses successive approximations to obtain more accurate solutions to a linear system at each step. Consider a system of n linear equations Ax=b or

$$\sum_{j=1}^{n}a_{i,j}x_j=b_i\,,\quad i=1,2,\ldots\ldots\ldots,n$$

The pseudo code for the successive overrelaxation algorithm is given as follows:

Algorithm:

Input: omega, nmax, toll, err, iter.

Define: r=b-a*x0

```
        r0=norm(r)
        err= norm(r)
                        xold=x0

Choose an initial guess x0 to the solution x.
iter=0
     while err>toll & iter<nmax
iter=iter+1
     for i=1,2,-------,n
s=0
     for j=1,2,-------,i-1
s=s+a(i,j)x(j)
     end
     for j=i+1,i+2,-----,n
s=s+a(i,j)*xold(j)
     end
x(i)=omega*(b(i)-s)/a(i,j)+(1-omega)*xold(i)
```

```
    end
x=x(:)
xold=x
r=(b-a)*x
err=norm(r)/r0
    end
end
```

The “fsolve” function of MATLAB implements the successive overrelaxation method.

7. Numerical illustrations

In this section, we obtain the numerical results for the blocking probabilities of the new calls in cell A, cell B and cell C and the blocking probability of the calls in handoff area by using the MATLAB software. The performance measures are obtained by varying various parameters namely arrival rate λ_{nA}, λ_{nB}, λ_{nC} and service rate μ in Figs 1(a-b). The default parameters are set as N=10, Q=5, *l*=0.3, μ=0.5, λ_{nA}=0.2, λ_{nB} =0.3, λ_{nC} =0.4.

Figs 1(a,b) and 2(a,b) show the effect of the total number of channels N in cells A and cell B. In figs 1(a) and 2(a), it can be noticed that the blocking probabilities of new calls in cell A and B show increasing trends with the arrival rate λ_{nA} and λ_{nB}, respectively. Figs 1(b), 2(b) demonstrate the trends of blocking probability of handoff calls B_h by varying arrival rates λ_{nA} and λ_{nB}, respectively. Further, it is noticed that as we increase N, the blocking probability of handoff calls B_h decreases.

Figs 3(a,b) show the blocking probabilities of new and handoff calls in cell C on varying the new arrival rate λ_{nC} for different values of total number of channels N=10-14. We note that B_{nC} increases when λ_{nC} increases. It is easily observed from the graph that B_h initially decreases gradually and afterwards rapidly on increasing λ_{nC}.

Figs 4(a-c) exhibit the blocking probabilities of new calls in cell A, B and C for different values of service rate μ. The blocking probabilities of new calls B_{nA}, B_{nB} and B_{nC} decrease rapidly with the increases in μ.

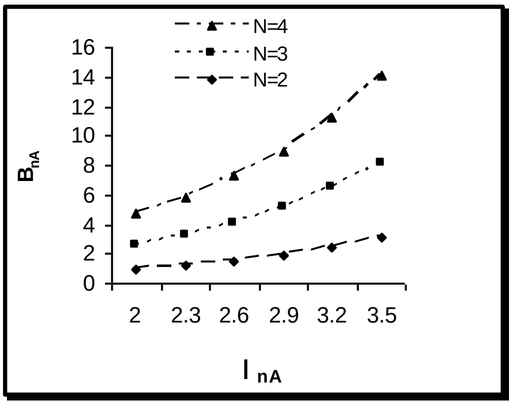
In conclusion, we can say that it offers the least blocking of the handoff calls as well as to the new calls can be achieved by the choice of suitable service rate. Numerical results provided demonstrate the computational tractability of the analytical results as well as give insight how the grade of service (GoS) of the system can be insured.

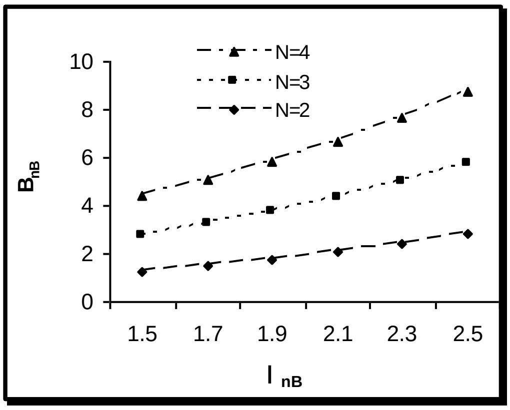
Conclusion

We presented the modeling and performance evaluation in the integrated ad hoc and cellular network system. The newness in the model is that we able to improve the performance in terms of performing experimental analysis on the preferred traffic model of priority scheme. The integrated system with the handoff area which is unstable, complicated and very important was modeled and analyzed. The novel approach is based on the development of model whose state spaces grow linearly with the number of calls that can be simultaneously in progress within a cell. We have obtained the probability vectors, which are further used to determine various performance measures.

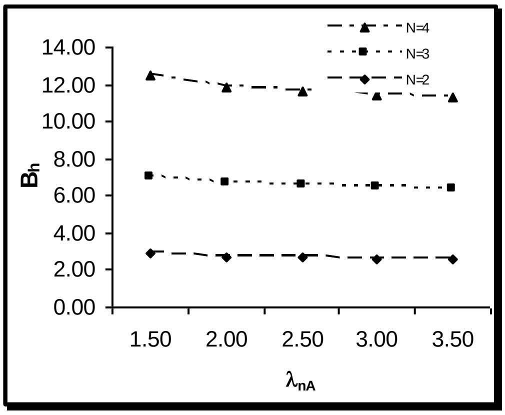
The suggested models are easy to employ and improves the performance of handoff calls but at the cost of new calls as the blocking of new calls increases. The selection of particular handoff calls depends on its implementation complexity and performance. The sensitivity analysis done may be helpful to system engineers during design and development phases. Though, limited number of allocated channels has been considered for simplicity still the model shows satisfactory results in some extend.

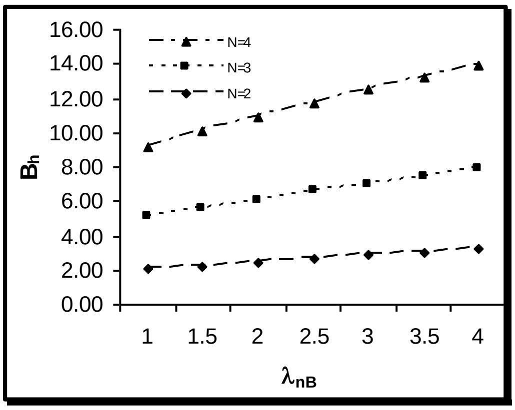
The result shows that the Successive Over-Relaxation method is more efficient than the other method. It can be considering their performance, using parameters as time to converge, storage and level of accuracy. Based on our investigation, the suggested model may be implemented to improve the grade of the service (GOS) of the real time system. Further study on the users mobility and presence of more callers in each cell with supplementary channels are in progress.





(a) (a)

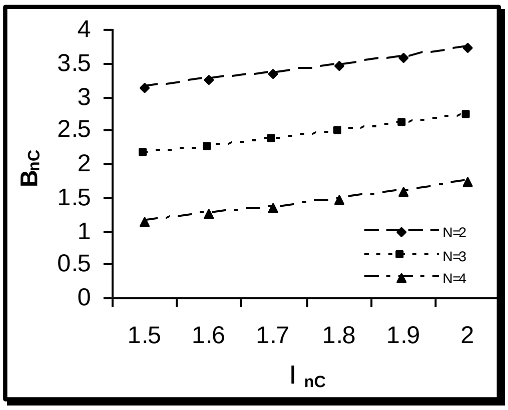




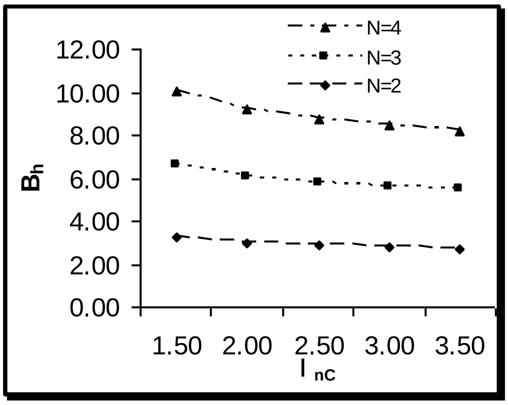
(b) (b)

Figs. 1(a,b): Blocking Probabilities of new and handoff calls in cell A by varying □$_{nA}$.

Figs. 2(a,b): Blocking Probabilities of new and handoff calls in cell B by varying □$_{nB}$.

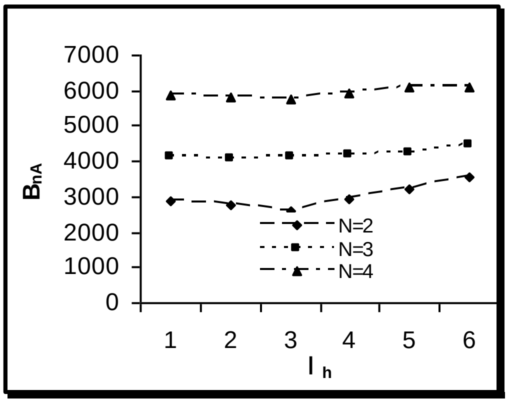


(a)

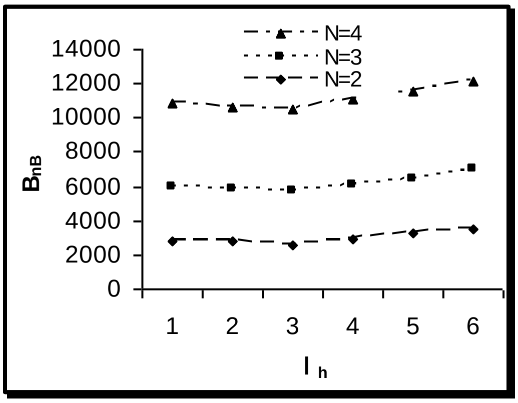


(b)

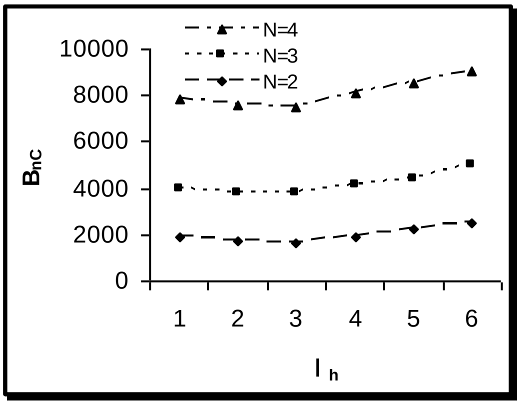
Figs. 3(a,b): Blocking Probabilities of new and handoff calls in cell C by varying □$_{nC}$.



(a)

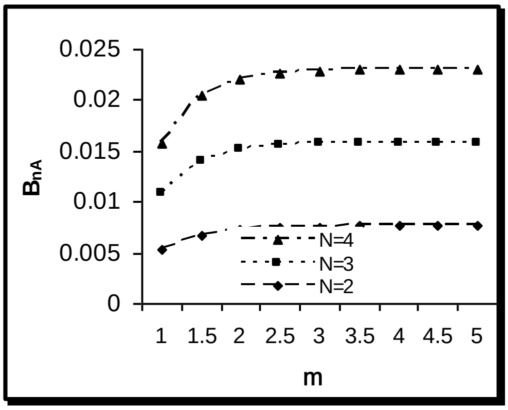


(b)



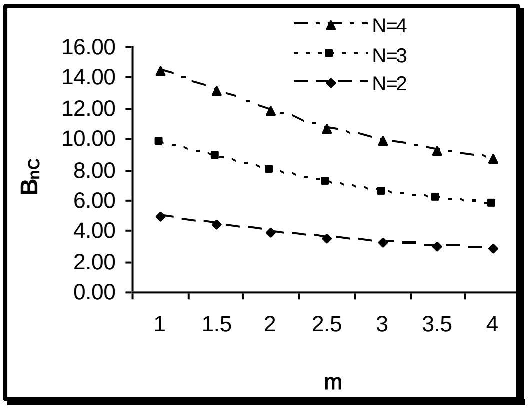
(c)

Figs. 4(a-c): Effect of blocking probabilities (a)B_{nA} (b)B_{nB} (c)B_{nC} in cells A,B and C by varying handoff arrival rate $\square_h$



(a)

(b)



(c)

Figs. 5(a-c): Effect of blocking probabilities (a)B_{nA} (b)B_{nB} (c)B_{nC} in cells A,B and C by varying service rate □

References

1. Banda, L. and Mzyece, M. (2014): Maximisation of correct handover probability and data throughput in vehicular networks, J. Mach. Mach. Commun., VOL. 1, No. 2, pp. 123–144 .
2. Bhattachary, S., Gupta, H. M. and Kar, S. (2008): “Traffic model and performance analysis of cellular mobile systems for general distributed handoff traffic and dynamic channel allocation,” Veh. Tech. IEEE Trans., vol. 57, no. 6, pp. 3629-3640.
3. Cruz-Perez, F. A., Toledo-Marin, R. and Hernandez-Valdez, G. (2011): Approximated mathematical analysis methods of guard channel based call admission control in cellular networks, Cellular. Net. Positioning., Perf. Analysis., Reliability, A. Melikov, Ed., Intech.
4. Dharmaraja, S., Trivadi, K., Logothetis, D. (2003): Performance modeling of wireless networks with generally distributed handoff interarrival times, Comput. Commun., Vol. 26, pp. 1747-1755.
5. Fang, Y. and Chlamtac, I. (2002): Analytical generalized results for handoff probability in wireless networks, IEEE Trans. Commun., Vol. 50, No. 3, pp. 396-399.
6. Garcia, D., Martinez, J. and Pla, V. (2005): Admission control policies in multiservice cellular networks, optimum configuration and sensitivity, wireless systems and mobility in next generation internet, Gabriele Kotsis and Otto Spaniol (eds.), Lecture Notes in Comp. Sci. (LNCS), Vol. 3427, pp. 121-135.
7. Guerin, R. (1998): Queueing blocking system with two arrival streams and guard channels, IEEE Trans. Commun., Vol. 36, pp. 153-163.
8. Guizani, S. (2016): Relay attack concerns in wireless ad hoc, sensors and RFID networks, Wireless commun. Mob. Comput., Vol 16, No. 11, PP 1431-1435.
9. Halgamuge, M. N., Ramamohanarao, K., Zukerman, M. and Vu, H. L. (2011): Handoff optimization using hidden markov model, Signal. Processing. Letters., IEEE Vol. 18, No. 7, pp. 411-414.
10. Hussain, M. A. (2015): Deployment of Mobile Adhoc network ticket based QOS routing protocol for healthcare, J. Sci. Tech., Vol. 8, No. 15, pp.1-5.
11. Jain, M. (2000): Prioritized channel assignment in mixed media cellular radio system, IETE Jour. Research., Vol. 46, No. 4, pp. 187-192.
12. Kim, H., Melikov, A., Fatlakhova, M. and Kim, C.S. (2012): An analytical approach to the analysis of guard channel based call admission control in wireless networks, J. Applied. Math., Vol. 2012, pp. 14.
13. Murillo-Perez, R. P., Rodriguez-Estrello, C. B. and Cruz-Perez, F.A. (2009): The impact of mobility on OFDMA-based cellular systems with reuse partitioning, IEEE Global Telecommun. Conf, ., pp. 1 – 7.
14. Pandey, K. K. , Baghel, N. and Sinha, S. K. (2013): Optinum Relay selection for energy efficient co operation Ad Hoc Networks, INT. J. Mod. Eng. Res., Vol. 43, No. 12, pp. 1082-108.
15. Remondo, D. and Niemegeers, I. G. (2003): Ad hoc networking in future wireless communications, Comput. Commun., Vol. 26, pp. 36-40.
16. Salamah, M. and Lababidi, H. (2005): Dynamically adaptive channel reservation scheme for cellular network, Comput. Net., Vol. 49, No. 6, pp. 787-796.
17. Saritha, V. and Viswanatham, V. M. (2014): Approach for channel reservation and allocation to improve quality of service in vehicular communications. Int’l J. IET Net. , Vol. 3, No. 2, pp. 150–159.

18. Sharma, G. and Purohit, G. N. (2011): Prioritized channel assignment to multimedia calls in wireless cellular networks, Int. J. Contemp. Math. Sciences., vol. 6, no. 33, pp. 1619-1628.
19. Sharma, V. and Jain, R. C. (2010): Multihop cellular networks: a review, Int. J. Eng. Sciences. Tech. , vol. 2, no. 11, pp. 6082-6028.
20. Soh, W. –S. and Kim, H. S. (2006): A predictive bandwidth reservation scheme using mobile positioning and road topology information, IEEE/ACM Trans. Net., vol. 14, pp. 1078-1091.
21. Wu, H. K. and Chuang, P. H. (2001): Dynamic QoS allocation for multimedia adhoc wireless networks, Mobile Net. Appli., Vol.6, pp. 377-384.
22. Wu, H., Qiao, C., De, S., and Tonguz, O. (2001): Integrated cellular and ad hoc relaying systems: iCAR, IEEE J. Select. Areas. Commun., Vol. 19, pp. 2105-2115.
23. Wu, H., Qiao, C., De, S., Yanmaz, E. and Tonguz, O. (2003): Queueing delay performance of the integrated cellular and ad hoc relaying systems, Proceed. IEEE Int. Conf. Commun., Vol. 19, pp. 923-927.
24. Wu, H., Qiao, C., De, S., Yanmaz, E. and Tonguz, O. (2005): Handoff performance of the integrated cellular and ad hoc relaying(iCAR) system, ACM Wireless. Net., Vol. 11, pp. 775-785.
25. Xhafa, A.E. and Tonguz, O.K. (2008): Handover Performance of priority schemes in cellular networks, Vehi. Tech. IEEE Trans., Vol. 57, No. 1, pp. 565-577
26. Yavuz, E. A. and Leung, V. C. M. (2007): Modeling channel occupancy times for voice traffic in cellular networks, Commun. IEEE Int. Conf., Vol. 24, No. 28, pp. 332-337. .
27. Zheng, Q., Hong, X. and Ray, S. (2004): Recent advances in mobility modeling for mobile adhoc network research, Proceed. Annual. Southeast. Reg. Conf., pp. 70-75.