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Plane Symmetric Magnetized Dark Energy Cosmological Model in f(R,T) Gravity

V. B. Raut[1] & S. A. Salve[2]
[1]*Department of Mathematics,*
Mungsaji Maharaj Mahavidyalaya, Darwha – 445 202, Maharashtra, India
[2]*Department of Mathematics,*
Jijamata Mahavidyalaya, Buldana – 443 001, Maharashtra, India
Corresponding Author – V. B. Raut
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Abstract:

In this study, we investigate a plane-symmetric cosmological model within the framework of $f(R,T)$ gravity, incorporating a magnetized dark energy component. The motivation behind this work stems from the growing interest in modified theories of gravity as viable alternatives to General Relativity (GR) in explaining the accelerated expansion of the universe. Our model considers a time-dependent deceleration parameter; the influence of magnetized dark energy is analyzed in detail, particularly in relation to its impact on the anisotropic behaviour of the universe. The behaviour of key cosmological parameters such as the Hubble parameter, energy density, and equation of state parameter is examined. Our findings suggest that $f(R,T)$ gravity provides a compelling framework for understanding the dynamical evolution of the universe, particularly in the presence of magnetized dark energy. The study emphasizes the importance of modified gravity theories in addressing the role of anisotropy in cosmic evolution.

Keywords: - Plane symmetric space-time, magnetized dark energy, $f(R,T)$ gravity.

Introduction:

Theoretical studies have revealed that a significant portion of the universe is composed of Dark Energy (DE) and Dark Matter (DM). Various DE models are characterized by their Equation of State (EoS) parameter, denoted as $\omega = \frac{p}{\rho}$. Observational astrophysical data suggest that this parameter is close to -1. When $\omega = -1$, it corresponds to the cosmological constant, which represents vacuum energy and is considered the simplest and most widely accepted candidate for DE [1-3]. If $\omega < -1$, it leads to the phantom DE model [4-6], whereas for the range $-1 < \omega < \frac{-1}{3}$, DE is described by the quintessence model [7-9]. The study of DE and its models within both General Relativity (GR) and Modified Theories of Gravity (MTG) has gained considerable research interest in recent years.

Several researchers have contributed to the exploration of DE in different cosmological settings. Akarsu and Kilinc [10] examined the anisotropy parameter of expansion for the Bianchi type-III model. Sharif and Zubair [11] analysed the Bianchi type-I universe under the influence of magnetized anisotropic DE with a variable EoS parameter. Kumar and Yadav [12] investigated a spatially homogeneous and anisotropic Bianchi type-V universe filled with DE under the assumption of minimal interaction, applying a specific law of variation for the Hubble parameter, and concluded that DE is dominant in the present cosmic epoch. Additionally, Amirhashchi *et al*. [13] and Pradhan *et al*. [14] proposed DE models in an anisotropic Bianchi type-VI space-time by considering constant and variable deceleration parameters, respectively. Saha and Yadav [15] derived exact solutions for a locally rotationally symmetric (LRS) Bianchi type-II DE model, illustrating the universe's transition from an initial decelerating phase to its current accelerating phase. Pawar *et al*. [16] studied magnetized DE cosmological models with a time-dependent cosmological term within the framework of Lyra geometry.

Among various modifications to Einstein's theory, $f(R,T)$ gravity [17] has been attracting significant attention. In this theory, the gravitational Lagrangian is defined as an arbitrary function of the Ricci scalar R and the trace of the energy-momentum tensor T. Sharif and Zubair [18] found that equilibrium thermodynamics is not feasible in $f(R,T)$ gravity. Katore *et al*. [19, 20] examined cosmological models incorporating DE within this framework. Rao and Neelima [21] formulated a Bianchi type-VI0 perfect fluid model using this theory. Chandel and Ram [22] generated a new set of field equation solutions for an anisotropic Bianchi type-III cosmological model with perfect fluid in $f(R,T)$ gravity. Chaubey *et al*. [23] further developed Bianchi-type cosmological models within the same framework. More recently, Sahoo *et al*. [24] investigated an axially symmetric space-time containing a perfect fluid source, while Chirde & Sheikh [25, 26] explored non-static plane symmetric space-time filled with DE, as well as the LRS Bianchi type-I universe featuring both decelerating and accelerating phases. Bhoyar *et al*. [27] examined a non-static plane symmetric cosmological model with magnetized anisotropic DE using a hybrid expansion law. Recent studies have delved into the interplay between anisotropic dark energy and $f(R,T)$ gravity, offering fresh insights into the universe's accelerated expansion and anisotropic characteristics. In a notable investigation, researchers examined a spatially homogeneous and anisotropic Bianchi type-I space-time filled with a perfect fluid within the $f(R,T)$ gravity framework. By adopting a specific functional form, $f(R,T) = R + 2f(T)$, with $f(T) = \alpha T + \beta T^2$, they derived exact solutions to the gravitational field equations. Assuming a hybrid expansion law for the average scale factor, the study elucidated various cosmological parameters, shedding light on the universe's anisotropic evolution and the role of dark energy in this modified gravity context [28].

Another significant contribution explored the energy conditions within $f(R,T)$ gravity against an anisotropic background. Focusing on a spatially homogeneous and anisotropic Bianchi type-VI0 universe, the authors derived the modified field equations pertinent to $f(R,T)$ gravity. Their analysis of the energy conditions provided constraints on the model parameters, offering deeper comprehension of anisotropic dark energy's influence on cosmic evolution within this theoretical framework [29]. Collectively, these studies underscore the versatility of $f(R,T)$ gravity in modeling anisotropic cosmological scenarios, enhancing our grasp of dark energy's anisotropic properties and their implications for the universe's dynamic behavior.

Motivated by the findings from this studies, this paper explores the properties of the plane symmetric space-time in the presence of magnetized anisotropic DE within the framework of $f(R,T)$ gravity. This paper is organized as follows: In section 2, we describe the brief review of $f(R,T)$ gravity. In section 3, we discussed the solution of the field equations. Sections 4, deals with some cosmological kinematical and physical parameters. Finally conclusions are summarized in the last section 5.

Formation of $f(R,T)$ Gravity:

We assume that the action for MTG of the following form (Harko *et al.* (2011))

$$s = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}\, d^4x + \int L_m \sqrt{-g}\, d^4x \quad , \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R, and T be the trace of the stress energy tensor of the matter T_{ij} and L_m is the matter Lagrangian density. We define the stress energy tensor of matter as

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}}, \tag{2}$$

and its trace by $T = g^{ij}T_{ij}$ respectively. By assuming that L_m of matter depends only on the metric tensor components g_{ij}, and not on its derivatives, we obtain

$$T_{ij} = g_{ij}L_m - 2\frac{\partial L_m}{\partial g^{ij}}. \tag{3}$$

Now by varying the action S of the gravitational field with respect to the metric tensor components g^{ij}, we obtain the field equations of $f(R,T)$ gravity as

$$f(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + f_R(R,T)\left(g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j\right) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\theta_{ij}, \tag{4}$$

where

$$\theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lk}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}}, \tag{5}$$

where $f_R = \frac{\delta f(R,T)}{\delta R}$, $f_T = \frac{\delta f(R,T)}{\delta T}$ and ∇_i is the covariant derivative and T_{ij} is the standard matter energy momentum tensor derived from the Lagrangian L_m. It may be noted that when $f(R,T) \equiv \varphi(R)$ the equation (4) yields the field equation of $f(R)$ gravity.

The problem of the perfect fluids described by an energy density ρ, pressure p and four velocities u^i is complicated since there is no unique definition of the matter Lagrangian. However, here we assume that the stress energy tensor of the matter is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij}, \tag{6}$$

and the matter Lagrangian can be taken as $L_m = -p$ and we have

$$u^i u_i = 1. \tag{7}$$

With the use of equation (5) we obtain for the variation of stress-energy tensor of perfect fluid is

$$\theta_{ij} = -2T_{ij} - p g_{ij}. \tag{8}$$

Generally, the field equation also depends through the tensor θ_{ij} and on the physical nature of the matter field. Hence in case of $f(R,T)$ gravity depending on the nature of the matter source, we obtain several theoretical models corresponding to each choice of $f(R,T)$. Assuming,

$$f(R,T) = R + 2f(T), \tag{9}$$

as a first choice where $f(T)$ is an arbitrary function of the trace of stress-energy tensor of matter, we get the gravitational field equations of $f(R,T)$ gravity from equation (4) as

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\theta_{ij} + f(T)g_{ij}, \tag{10}$$

where the prime denotes differentiation with respect to the argument. Using equation (8), above equation (10) become

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij}. \tag{11}$$

Field equations and its solution:

The line element of plane symmetric space-time is given by

$$ds^2 = dt^2 - A^2dx^2 - B^2(dy^2 + dz^2) \quad , \tag{12}$$

where the metric potentials A and B be the functions of time t only.

We do not expect the fluids to have bulk motion on cosmological scales, hence we assume that the fluid is co-moving i.e.

$u^i u_i = 1$. The energy momentum tensor for anisotropic dark energy is given by

$$T_j^i = diag[\rho, -p_x, -p_y, -p_z]$$
$$= diag\ [1, -w_x, -w_y, -w_z]\ \rho, \qquad (13)$$

where ρ is the energy density of the fluid, p_x, p_y, p_z and w_x, w_y, w_z are the directional pressure and EoS parameters of the fluid respectively along x, y, z axes respectively.

However, the true nature of Dark Energy (DE) remains an enigma, and there is no fundamental or observational justification for restricting DE to exhibit only isotropic pressure. Given that the fluids are co-moving, DE can also produce anisotropic pressure if its pressure is assumed to be proportional to its energy density. Consequently, the Equation of State (EoS) parameter of the DE fluid becomes direction-dependent. Therefore, the energy-momentum tensor for a magnetized DE fluid can be expressed as follows:

$$T_i^j = diag[\rho + \rho_B, -p_r + \rho_B,\ -p_\theta - \rho_B,\ -p_\varphi - \rho_B]$$
$$= diag[\rho + \rho_B, -w\rho + \rho_B, -(w + \delta)\rho - \rho_B, -(w + \delta)\rho - \rho_B], \qquad (14)$$

where δ be the deviation from the free EoS parameter (hence the deviation free pressure) on x -axis and y -axis and ρ_B stands for energy density of magnetic field.

When considering the magnetized DE source given in equation (14), the field equations (11) associated with the metric (12) result in the following set of linearly independent differential equations:

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} = \rho[(8\pi + 2\mu)w - \mu(1 - 3w - 2\delta)] - (8\pi + 2\mu)\rho_B - 2\mu p, \qquad (15)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = \rho[(8\pi + 2\mu)(w + \delta) - \mu(1 - 3w - 2\delta)] + (8\pi + 2\mu)\rho_B - 2\mu p, \qquad (16)$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} = -\rho(8\pi + 2\mu) - \mu\rho(1 - 3w - 2\delta) - (8\pi + 2\mu)\rho_B - 2\mu p. \qquad (17)$$

where the overhead dot denotes differentiation with respect to time t.

We have the following equation from the Bianchi identity,

$$\dot{\rho} + (1 + \omega)\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\rho + \left(2\delta\frac{\dot{B}}{B}\right)\rho = 0$$

and $\dot{\rho}_B + 4\frac{\dot{B}}{B}\rho_B = 0.$ (18)

The field equations presented in the equations (15) to (17), involve complex nonlinear differential equations due to the modified gravitational framework incorporating both the Ricci scalar R and the trace of the energy-momentum tensor T. The presence of additional terms, such as μ, ρ_B and directional pressure anisotropy parameters, further complicates their direct analytical solutions. To simplify these equations and make them more tractable, we impose the condition $w + \delta = 0$. This assumption effectively eliminates the dependency on δ, reducing the number of independent variables. Physically, this implies a specific relationship between the equation of state parameter w and the anisotropic deviation δ, ensuring that any anisotropic pressure variation is counterbalanced by the equation of state parameter.

By applying this condition, the field equations can be significantly simplified, this assumption is particularly useful in studying cosmological models where anisotropic effects are minimal or where a symmetric evolution of the universe is desired. With this condition the physical parameters such as energy density, equation of state parameter and the deviation from equation of state parameter are respectively observed as

$$\rho = \frac{1}{(8\pi + 2\mu)}\left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\dot{B}^2}{B^2} - \frac{1}{B^2}\right), \qquad (19)$$

$$\omega = -1 + \frac{1}{(8\pi + 2\mu)\rho}\left(2\frac{\ddot{B}}{B} - 2\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right), \qquad (20)$$

$\delta = 1 - \frac{1}{(4\pi+\mu)\rho}\left(\frac{\ddot{B}}{B} - \frac{\dot{A}}{A}\frac{\dot{B}}{B}\right).$ (21)

Finally, here we have three differential equations (15)-(17) with six unknowns namely $A, B, w, \rho, \delta, \rho_B$. We define some physical quantities of the space-time as

Average scale factor (a) and volume (V) respectively as

$a = \sqrt[3]{AB^2}$ $V = a^3$,. (22)

The generalized mean Hubble parameter (H) is of the form

$$H = \frac{1}{3}\left(H_x + H_y + H_z\right), \quad (23)$$

where H_x, H_y, H_z are the directional Hubble parameter in the direction of x, y and z -axis respectively.

Using equations (22) and (23), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(H_x + H_y + H_z\right) = \frac{\dot{a}}{a}. \quad (24)$$

The expansion scalar (θ) and shear scalar (σ^2) are defined as follows

$$\theta = u^{\mu}_{;\mu} = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right), \quad (25)$$

To find the solution of the field equation we need the extra conditions. Hence first we consider

1) The relation between the metric potentials as

$$A = B^n, \quad (26)$$

and

2) A time varying deceleration parameter

Incorporating a time-varying deceleration parameter into cosmological models is essential for accurately depicting the universe's dynamic evolution, particularly its transition from deceleration to acceleration. This parameter, denoted as $q(t)$, quantifies the rate of change of the cosmic expansion and is defined by the relation $q = -\frac{a\ddot{a}}{\dot{a}^2}$, where $a(t)$ represents the scale factor of the universe. A time-dependent $q(t)$ allows for a more nuanced modeling of the universe's expansion history, accommodating periods of both acceleration and deceleration. In the context of modified gravity theories, such as $f(R,T)$ gravity. Employing a time-varying deceleration parameter has yielded significant characteristics. For instance, Tiwari *et al.* [30] investigated a locally rotationally symmetric Bianchi type-I universe within $f(R,T)$ gravity, adopting a periodically varying deceleration parameter. Their findings demonstrated that this approach effectively captures the observed cosmic acceleration and provides a coherent description of the universe's evolution. Similarly, Sahoo *et al.* [31] explored the implications of a periodic deceleration parameter in $f(R,T)$ gravity, revealing oscillatory behaviors in cosmological parameters that align with observational data. These studies underscore the importance of integrating a time-varying deceleration parameter in modified gravity frameworks to enhance our comprehension of cosmic dynamics. In this case, we have considered the time special form of time varying deceleration parameter [32, 33]

$$q = -1 + \frac{\beta}{1+a^{\beta}}, \quad (27)$$

where $\beta > 0$ is a constant. Consequently, the Hubble's parameter is

$$H = a_1\left(a^{-\beta} + 1\right), \quad (28)$$

where a_1 is an integrating constant. Again, integrating the above equation, we have

$$a = \left(e^{a_1\beta t} - 1\right)^{\frac{1}{\beta}}. \quad (29)$$

Hence, the metric potentials are obtained as

$$A = \left(e^{a_1\beta t} - 1\right)^{\frac{3n}{\beta(n+2)}}. \quad (30)$$

$$B = \left(e^{a_1\beta t} - 1\right)^{\frac{3}{\beta(n+2)}}. \quad (31)$$

Physical and Kinematical behavior of the model:

From the equation of conservation, we get the energy density of magnetic field as

$$\rho_B = C\left(e^{a_1\beta t} - 1\right)^{\frac{-12}{\beta(n+2)}}, \tag{32}$$

where C be the constant of integration.

In our analysis, we found that the energy density of the magnetic field in the presence of dark energy exhibits a positive yet decreasing behavior over time. This indicates that while the magnetic field contributes positively to the total energy density of the universe, its influence diminishes as the universe evolves. Such a trend aligns with the expectation that as cosmic expansion progresses, the intensity of the magnetic field weakens due to the stretching of field lines in an expanding spacetime. This behavior is clearly seen in the Fig. 1.

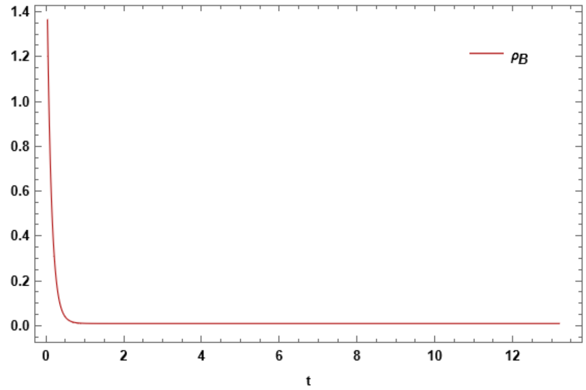

Fig. 1: The behavior of the energy density of the magnetic field versus cosmic time.

The decreasing nature of magnetic energy density suggests that its role in the late-time dynamics of the universe becomes less significant, reinforcing the dominance of dark energy in driving cosmic acceleration. This behavior is crucial in understanding the interaction between magnetized anisotropic dark energy and the evolution of large-scale structures, providing the possible dissipation mechanisms of cosmic magnetic fields in modified gravity frameworks.

The energy density of the derived model is observed as

$$\rho = \frac{\left(-1+e^{a1t\beta}\right)^{-2-\frac{6}{(2+n)\beta}}}{2(2+n)^2(4\pi+\mu)}\left(\begin{array}{c}\left(-(2+n)^2 + e^{2a1t\beta}\left(9a1^2\left(-1+e^{a1t\beta}\right)^{\frac{6}{(2+n)\beta}}(-1+n)n - (2+n)^2\right)\right. \\ \left. -e^{a1t\beta}(2+n)\left(-2(2+n) + 3a1^2\left(-1+e^{a1t\beta}\right)^{\frac{6}{(2+n)\beta}}(1+n)\beta\right)\right)\end{array}\right)$$

In our analysis, we observed that the energy density of the model in the presence of dark energy exhibits a positive but decreasing behavior over time. This behavior is clearly seen in the Fig. 2.

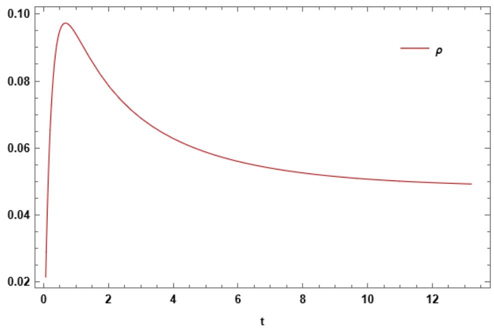

Fig. 2: The behavior of the energy density of the model versus cosmic time.

This implies that while dark energy maintains a dominant presence in the cosmic framework, its energy density gradually declines as the universe expands. Such a trend is consistent with many dark energy models, where the dilution of energy density occurs due to the continuous expansion of spacetime. The positive yet decreasing nature of the energy density suggests a dynamic evolution of dark energy, potentially aligning with quintessence or other time-dependent dark energy scenarios.

This behavior plays a significant role in understanding the late-time acceleration of the universe and its future evolution. Moreover, it provides how dark energy interacts with other cosmic components, such as matter and radiation, within the framework of modified gravity theories, further enriching our comprehension of the universe's accelerating expansion.

The equation of state parameter of the derived model is observed as

$$\omega = -1 - \frac{3\mathrm{a1}^2 e^{\mathrm{a1}t\beta}(3e^{\mathrm{a1}t\beta}(-1+n)+(2+n)\beta)}{(-1+e^{\mathrm{a1}t\beta})^2(2+n)^2(4\pi+\mu)}$$

In our analysis, we found that the equation of state parameter of the dark energy model exhibits a negative and decreasing behavior as the universe expands. This behavior is clearly seen in the Fig. 3.

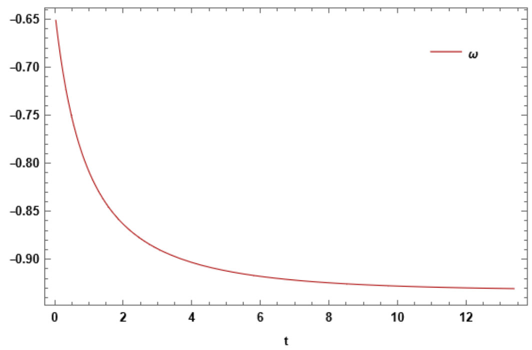

Fig. 3: The behavior of equation of state parameter of the model versus cosmic time.

At the initial stage of expansion, when $t = 0$, the equation of state parameter is approximately -0.65, indicating that dark energy initially behaves like a quintessence-type component. As the universe continues to expand, the equation of state parameter gradually decreases, reaching around -0.90, signifying a transition toward a more dominant dark energy influence. Eventually, at the present epoch, the equation of state parameter converges to -1, which corresponds to the cosmological constant (Λ) and represents the well-established vacuum energy scenario. This evolution suggests that dark energy in the model dynamically evolves from a quintessence-like phase to a cosmological constant phase, driving the accelerated expansion of the universe. Such behavior is consistent with observational data and provides a deeper understanding of the nature and evolution of dark energy within the framework of modified gravity theories.

The deviation from the free equation of state parameter of the derived model is observed as

$$\delta = 1 + \frac{3\mathrm{a1}^2 e^{\mathrm{a1}t\beta}(3e^{\mathrm{a1}t\beta}(-1+n)+(2+n)\beta)}{(-1+e^{\mathrm{a1}t\beta})^2(2+n)^2(4\pi+\mu)}$$

In our analysis, we observed that the deviation from the free equation of state parameter in the dark energy model exhibits a positive and increasing behavior as the universe expands.

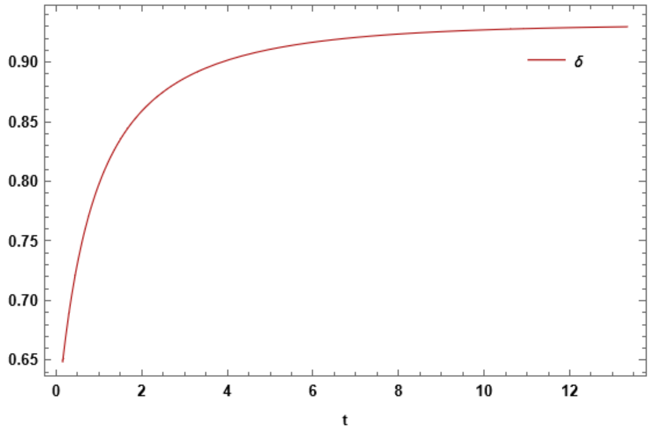

Fig. 4: The behavior of equation of state parameter of the model versus cosmic time.

Initially, at $t = 0$, its value is approximately 0.65, indicating a deviation from the standard equation of state parameter. As the universe expands, this deviation gradually increases, reaching around 0.85, suggesting a growing departure from the conventional dark energy behavior. Eventually, at the present epoch, it approaches 1, indicating that the dark energy model evolves toward a state with a stronger deviation from a free equation of state parameter. This increasing behavior highlights the dynamical nature of dark energy, suggesting that it does not remain constant over cosmic time but instead evolves in response to the expansion of the universe.

Furthermore, the existence of this deviation suggests the presence of anisotropy in the cosmic evolution. In an isotropic universe, the equation of state parameter is expected to be uniform in all directions; however, a deviation from its free form implies directional dependence, leading to anisotropic pressure distribution. This anisotropy could arise due to the influence of primordial magnetic fields, anisotropic expansion, or interactions between dark energy and other cosmic components. The persistence of such anisotropy throughout cosmic evolution could provide valuable insights into the underlying nature of dark energy and its role in the large-scale structure formation of the universe. Understanding this anisotropic behavior is crucial in refining cosmological models and testing the validity of modified gravity theories.

The Hubble's parameter and the expansion scalar of the model is observed as

$$\theta = 3H = 3\left\{\mathrm{a1}\left(1 + \frac{1}{-1 + e^{\mathrm{a1}t\beta}}\right)\right\}$$

In our analysis, we found that the expansion scalar is a function of time and exhibits a positive decreasing behavior as the universe expands (See Fig.5, *blue curve*). The expansion scalar quantifies the rate at which the volume of a given region in the universe changes over time. Initially, at the early stages of cosmic evolution, the expansion scalar has a higher value, indicating a rapid expansion. However, as the universe evolves, the expansion rate gradually slows down, leading to a decreasing expansion scalar. This behavior is consistent with the standard cosmological model, where the expansion rate was much higher in the early universe due to dominant radiation and matter contributions, but as dark energy takes over in the later stages, the expansion becomes more controlled. The positive nature of the expansion scalar ensures that the universe continues to expand.

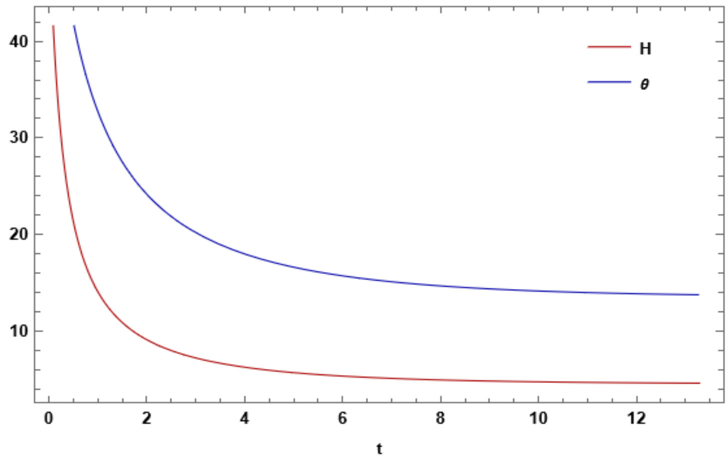

Fig. 5: The behavior of expansion scalar (blue curve) and the Hubble's parameter (red curve) of the model versus cosmic time.

Our analysis also indicates that the (See Fig.5). is a function of time and follows a similar positively decreasing trend as that of expansion scalar as the universe expands (See Fig.5 *red curve*). The Hubble parameter represents the rate of expansion of the universe at a given time and is crucial for determining cosmic evolution. In the early universe, the Hubble parameter had a higher value, reflecting the rapid expansion following the Big Bang. As time progresses, the Hubble parameter decreases, implying a slower expansion rate, although it remains positive, ensuring continuous cosmic expansion. This decline is attributed to the transition from a radiation and matter-dominated universe to a dark energy-dominated phase. The decreasing nature of the Hubble parameter also suggests a gradual approach toward a more stable expansion rate in the late universe, potentially converging to a constant value in the case of a de Sitter-like future. This behavior plays a fundamental role in understanding cosmic acceleration and the influence of dark energy within the framework of general relativity and modified gravity theories.

The deceleration parameter of the model is observed as

$$q = -1 + \frac{\beta}{1 + ((-1 + e^{a1t\beta})^{3/\beta})^{\beta}}$$

In our analysis, we found that the deceleration parameter (q) is a function of time and exhibits a negatively decreasing behavior as the universe expands (See Fig. 6). Initially, at $t = 0$, the deceleration parameter starts at approximately -0.80, indicating that the universe is already in an accelerated expansion phase. As time progresses and the universe expands, q continues to decrease and asymptotically approaches -1 at $t \to \infty$. This trend suggests a transition toward a de Sitter-like expansion, where dark energy dominates completely, leading to an exponential cosmic acceleration.

Physically, the deceleration parameter characterizes whether the universe is accelerating or decelerating. A positive q would indicate a decelerating universe, dominated by matter or radiation, while a negative q confirms an accelerated expansion driven by dark energy. The observed behavior of q in our study aligns with current cosmological models where dark energy, modeled as a cosmological constant (Λ), gradually dominates the universe's dynamics. The approach of q toward -1 suggests that the universe may eventually evolve into a state of perpetual acceleration, resembling a de Sitter expansion scenario. This result is crucial in understanding the fate of the universe and supports the idea that dark energy plays a fundamental role in driving cosmic acceleration.

Discussion and Concluding Remark:

Our investigation into the plane-symmetric magnetized dark energy cosmological model within the framework of $f(R,T)$ gravity provides significant insights into the evolution of the universe. The study highlights the role of modified gravity in explaining the late-time accelerated expansion of the cosmos and presents a compelling case for incorporating anisotropic effects into cosmological model by considering a time-dependent deceleration parameter.

One of the key findings of our study is the behavior of the energy density of dark energy, which remains positive while exhibiting a decreasing trend as the universe expands. This result is consistent with the idea that dark energy dominates the late-time dynamics of the cosmos, driving its accelerated expansion. Furthermore, the equation of state parameter for dark energy is observed to be negative and decreasing over time, asymptotically approaching -1. This behavior aligns with the ΛCDM model and supports the hypothesis that dark energy behaves similarly to a cosmological constant at present times.

The deceleration parameter, which is a crucial indicator of cosmic expansion dynamics, also shows significant variation over time. At early times, its value is around -0.80, indicating a phase of decelerated expansion. However, as the universe evolves, the deceleration parameter gradually decreases and asymptotically reaches -1, signifying an eventual transition to a de Sitter-like expansion. This behavior confirms that our model successfully captures the essential features of cosmic evolution observed in modern astrophysical data.

Another crucial aspect of our study is the examination of the deviation from a free equation of state parameter. Our analysis indicates that this deviation exhibits a positive increasing behavior at the initial stages of cosmic evolution, with its value close to 0.65 at $t = 0$. As the universe expands, this deviation increases up to 0.85 and eventually approaches 1 at present times. This finding suggests that deviations from the standard equation of state may play a crucial role in determining the anisotropic nature of the early universe. Such anisotropies are expected to leave imprints on cosmic structures and may be observable through large-scale anisotropic distributions in the cosmic microwave background radiation.

The presence of a magnetized dark energy component adds another layer of complexity to the model. Our results indicate that an anisotropic magnetic field can influence the dynamics of cosmic expansion by modifying the evolution of cosmological parameters. In particular, the introduction of a magnetized component enhances the anisotropic nature of the model, distinguishing it from standard isotropic dark energy models. This observation aligns with several previous studies that have suggested that early-universe anisotropies, potentially induced by primordial magnetic fields, could have played a role in shaping cosmic structures.

Furthermore, our analysis of the expansion scalar and the Hubble parameter reveals a consistent pattern. Both parameters are found to be time-dependent and exhibit a decreasing trend as the universe expands. The expansion scalar, which provides a measure of the rate of expansion, shows a gradual decrease over time, indicating a shift from an initial rapid expansion phase to a more stable accelerated expansion. Similarly, the Hubble parameter follows a decreasing trend, further reinforcing the notion that the universe is evolving towards a steady expansion rate governed by dark energy dominance.

Overall, our study reinforces the viability of $f(R,T)$ gravity as an alternative framework for explaining cosmic acceleration. The presence of anisotropic dark energy, coupled with a magnetized field, introduces novel features that distinguish this model from standard isotropic cosmologies. Our findings suggest that anisotropic effects should not be neglected in cosmological modeling, as they may have played a crucial role in shaping the universe's large-scale structure.

In conclusion, our study provides a comprehensive analysis of a plane-symmetric cosmological model within $f(R,T)$ gravity, emphasizing the significance of magnetized dark energy and anisotropic effects. The results obtained in this research contribute to the ongoing efforts to develop alternative cosmological models that can address the limitations of the standard ΛCDM paradigm. As future observational techniques become more sophisticated, models incorporating anisotropic dark energy and modified gravity theories may offer a more complete understanding of the fundamental nature of our universe.

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