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Original Article

COUPLED COINCIDENCE BEST PROXIMITY POINT THEOREMS IN INTUITIONISTIC FUZZY METRIC SPACES WITH DIRECTED GRAPH AND PARTIAL ORDERING

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Abstract:

This paper introduces and establishes novel coupled coincidence best proximity point theorems within the framework of intuitionistic fuzzy metric spaces (IFMS), enriched with the structure of a directed graph and a partial ordering. By harmonizing three distinct abstract structures IFMS, graph theory, and order theory we generalize and extend several pivotal fixed point and best proximity point results from the existing literature. The primary theorems are proven under a novel contraction condition that amalgamates the intuitionistic fuzzy metric, the properties of a directed graph (denoted by G), and a partial order (⪯). A significant outcome demonstrates that under specific G-continuity, G-regularity, and transitivity conditions, the sequences generated by the mixed monotone property converge to a coupled coincidence best proximity point. As a corollary, a new coupled fixed point theorem in complete IFMS is derived. To validate the theoretical findings and illustrate their superiority over existing results, a non-trivial example is provided. The paper concludes with a discussion on potential applications in nonlinear integral equations and future research directions.

Keywords: Intuitionistic Fuzzy Metric Space; Best Proximity Point; Coupled Coinci- Dence Point; Directed Graph; Partial Order; Mixed Monotone Property; G-Continuity.

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1. Introduction:

Fixed point theory constitutes a cornerstone of nonlinear functional analysis, providing powerful tools for establishing the existence and uniqueness of solutions to differential equations, integral equations, and variational inequalities. The celebrated Banach Con- traction Principle [3] has been generalized along numerous avenues, including the expan- sion of the underlying space (e.g., to metric, fuzzy metric, and intuitionistic fuzzy metric spaces) and the relaxation of contraction conditions.

The coupled fixed point concept, pioneered by Guo and Lakshmikantham [7] and later refined by Bhaskar and Lakshmikantham [4], addresses equations of the form $F(x, y) = x$ and $F(y, x) = y$. This framework is particularly apt for investigating mixed monotone operators in ordered spaces, finding applications in periodic boundary value problems.

Simultaneously, best proximity point theory emerged to solve minimization problems of the form $\text{dist}(x, Tx) = \text{dist}(A, B)$ when a mapping $T : A \to B$ does not necessarily have a fixed point. It ensures the existence of an optimal approximate solution. Sankar Raj [12] extended this notion to coupled best proximity points for mappings $F : A \times A \to B$.

In parallel, the fuzzy metric space [9] and its generalization, the intuitionistic fuzzy metric space [11], which incorporates both membership and non-membership degrees, have provided a more nuanced framework for modeling uncertainty and imprecision. Fixed point results in these spaces have been prolific (e.g., [6, 1]).

Recently, the unification of metric fixed point theory with graph theory, initiated by Jachymski [8], has opened a new paradigm. By replacing the global metric condition with one defined along the paths of a graph, results become more flexible. This approach was combined with partial orders by Nieto and Rodr´ıguez-López [10], and later with best proximity points by Alghamdi et al. [2].

Research Gap and Motivation:

Despite these parallel developments, a significant gap exists in the amalgamation of these three potent structures intuitionistic fuzzy metrics, graph theory, and partial or- ders specifically for *coupled coincidence best proximity point* problems. This unified approach can model complex systems where:

- Uncertainty is inherent (handled by IFMS),
- The interaction between points is not universal but follows a specific relational structure (modeled by a directed graph),
- An order relation exists on the space (partial order).

Contribution:

This paper bridges this gap. We define coupled coincidence best proximity points in the context of IFMS endowed with a directed graph G and a partial order $\preceq$. We establish comprehensive existence and convergence theorems under a novel $(G, \preceq, \varphi)$ contraction condition. Our results:

- Generalize the coupled best proximity point theorems of Sankar Raj [12] and Choud- hury et al. [5] to IFMS.
- Extend the graph contraction principle of Jachymski [8] and coupled fixed point theo- rems in ordered metric spaces [4] to the coupled best proximity setting in IFMS.

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- Unify and extend several results from the literature by toggling the presence of the graph, the order, or the best proximity condition.

The paper is structured as follows: Section 2 recaps essential definitions. Section 3 presents the main coupled coincidence best proximity point theorems. Section 4 deduces a coupled fixed point theorem as a corollary. Section 5 provides a supporting example and discusses applications. Section 6 concludes and outlines future work.

2. Preliminaries:

Definition 2.1 ([1]). *A 5-tuple $(X, M, N, *, \diamond)$ is an **intuitionistic fuzzy metric space (IFMS)** if X is a nonempty set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm, and M, N are fuzzy sets on $X^2\times[0,\infty)$ satisfying for all $x,y,z \in X$, $s,t > 0$:*

i. $M(x,y,t)+N(x,y,t) \le 1$,
ii. $M(x,y,t) > 0$,
iii. $M(x,y,t) = 1$ *if and only if* $x = y$,
iv. $M(x,y,t) = M(y,x,t)$,
v. $M(x,y,t) * M(y,z,s) \le M(x,z,t+s)$,
vi. $M(x,y,\cdot) : [0,\infty) \to [0,1]$ *is continuous,*
vii. $N(x,y,t) > 0$, $N(x,y,t) = 0$ *if and only if* $x = y$,
viii. $N(x,y,t) = N(y,x,t)$,
ix. $N(x,y,t) \diamond N(y,z,s) \ge N(x,z,t+s)$,
x. $N(x,y,\cdot) : [0,\infty) \to [0,1]$ *is continuous.*

Here, (M,N) is called an intuitionistic fuzzy metric.

Definition 2.2 ([11]). *Let $(X,M,N,*,\diamond)$ be an IFMS.*

1. *A sequence $\{x_n\}$ **converges** to x if* $\lim_{n\to\infty} M(x_n,x,t) = 1$ *and* $\lim_{n\to\infty} N(x_n,x,t) = 0$ *for all* $t > 0$.
2. *It is a **Cauchy sequence** if for $\varepsilon > 0$, $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $M(x_n,x_m,t) > 1-\varepsilon$ and $N(x_n,x_m,t) < \varepsilon$ for all $n,m \ge n_0$.*
3. *The space is **complete** if every Cauchy sequence converges.*

Definition 2.3 ([8]). *Let X be a nonempty set and $G = (V(G),E(G))$ be a directed graph where $V(G) = X$ and $E(G)$ contains all loops. For $x,y \in X$, a **path** from x to y is a sequence $\{p_i\}_{i=0}^{N}$ with $p_0 = x$, $p_N = y$ and $(p_{i-1},p_i) \in E(G)$ for $i = 1,...,N$.*

Definition 2.4 ([4]). *Let $(X,\preceq)$ be a partially ordered set. A mapping $F : X\times X \to X$ has the **mixed monotone property** if for any $x,y \in X$,*

$$x_1,x_2 \in X, x_1 \preceq x_2 \implies F(x_1,y) \preceq F(x_2,y),\ y_1,y_2 \in X,\ y_1 \preceq y_2 \implies F(x,y_1) \succeq F(x,y_2).$$

Definition 2.5 ([12]). *Let A,B be nonempty subsets of a metric space (X,d) and $F : A\times A \to B$. A point $(p,q) \in A\times A$ is a **coupled best proximity point** if*

$$d(p,F(p,q)) = d(q,F(q,p)) = dist(A,B).$$

We adapt this for IFMS. Let A, B be nonempty subsets of an IFMS $(X,M,N,*,\diamond)$. Define the intuitionistic fuzzy distance between sets: For $t > 0$,

$$M(A,B,t) = \sup\{M(a,b,t) : a \in A, b \in B\},$$
$$N(A,B,t) = \inf\{N(a,b,t) : a \in A, b \in B\}.$$

Definition 2.6. *Let $A,B \subseteq X$ of an IFMS, $F : A\times A \to B$, and $g : A \to A$. A point $(p,q) \in A\times A$ is a **coupled coincidence best proximity point** of F and g if for all $t > 0$:*

$$M(gp,F(p,q),t) = M(A,B,t) \quad \text{and} \quad N(gp, F(p,q),t) = N(A,B,t),$$

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$M(gq,F(q,p),t) = M(A,B,t)$ and $N(gq, F(q,p),t) = N(A,B,t)$.

*If g is the identity map, it is a **coupled best proximity point**.*

Definition 2.7. *Let $(X,M,N,*,\diamond,G)$ be an IFMS with a directed graph G.*

- *$F : X \times X \to X$ is **G-continuous** at x if for any sequence $\{x_n\}$ with $(x_n, x_{n+1}) \in E(G)$ and $x_n \to x$, we have $F(x_n,y) \to F(x,y)$ and $F(y,x_n) \to F(y, x)$ for all y.*
- *(X, M, N, G) is **G-regular** if for any sequence $\{x_n\}$ with $(x_n, x_{n+1}) \in E(G)$ converging to x, we have $(x_n, x) \in E(G)$ for all n.*

3. Main Results:

Let $(X, M, N, *, \diamond)$ be a complete IFMS. Let A, B be nonempty closed subsets of X. Let $G = (V(G), E(G))$ be a directed graph with $V(G) = X$ and $\Delta = \{(x,x) : x \in X\} \subseteq E(G)$. Let $\preceq$ be a partial order on X. Define for each $t > 0$:

$A_0(t) = \{a \in A : \exists b \in B$ with $M(a,b,t) = M(A,B,t)$ and $N(a,b,t) = N(A,B,t)\}$.

We assume $A_0(t)$ is nonempty for all $t > 0$ and $F(A \times A) \subseteq B$. Let $\varphi : [0,1]\times[0,1] \to [0,1]$ be a function such that $\varphi(s,t) > \max(s,t)$ for $s,t \in (0,1)$.

Definition 3.1. *The mapping $F : A \times A \to B$ is a $(G,\preceq,\varphi)$-**contraction** if for all $x,y,u,v \in A$ with $(x,u),(y,v) \in E(G)$ and $x \preceq u$, $y \succeq v$, and for all $t > 0$:*

1. $\varphi(M(F(x, y), F(u, v), t), M(gx, gu, t)) \geq \min(M(gx,gu,t),M(gy,gv,t))$,
2. $\varphi(N(F(x, y), F(u, v), t), N(gx, gu, t)) \leq \max(N(gx,gu,t),N(gy,gv,t))$, *where $g : A \to A$ is a surjective mapping.*

Theorem 3.2. *Let $(X,M,N,*,\diamond,G,\preceq), A,B,F,g,\varphi$ be as above. Assume: (C1) F has the mixed $(G,\preceq)$-monotone property: For $x,y \in A$,*

$(x_1, x_2) \in E(G)$, $x_1 \preceq x_2 \implies (F(x_1, y), F(x_2,y)) \in E(G)$ and $F(x_1,y) \preceq F(x_2,y)$,
$(y_1,y_2) \in E(G)$, $y_1 \preceq y_2 \implies (F(x,y_2),F(x, y_1)) \in E(G)$ and $F(x,y_2) \preceq F(x,y_1)$.

(C2) There exist $x_0,y_0 \in A_0(t)$ such that $(gx_0, F(x_0, y_0)), (gy_0, F(y_0, x_0)) \in E(G)$, $gx_0 \preceq F(x_0, y_0)$, $gy_0 \succeq F(y_0,x_0)$.
(C3) F is $(G,\preceq)$-continuous, or (X,M,N,G) is G-regular and A is closed. (C4) g is compatible with the graph and order, and g(A) is complete. (C5) For each $t > 0$, $M(A, B,t) > 0$ and $N(A,B,t) < 1$.
Then, F and g have a coupled coincidence best proximity point in $A \times A$.
Proof Sketch.

Step 1: Construct Iterative Sequences. Using condition (C2), define sequences $\{x_n\}$, $\{y_n\}$ in A:

$gx_{n+1} = F(x_n,y_n)$,
$gy_{n+1} = F(y_n,x_n)$, for $n \geq 0$.

From (C1), (C2), and induction, we show $(gx_n, gx_{n+1}), (gy_n, gy_{n+1}) \in E(G)$, $gx_n \preceq gx_{n+1}$, $gy_n \succeq gy_{n+1}$.

Step 2: Prove Sequences are Cauchy. Using the $(G,\preceq,\varphi)$-contraction condition, we establish that for all $t > 0$,

lim $M(gx_n,gx_{n+1},t) = 1$ and lim $N(gx_n, gx_{n+1},t) = 0$.

$n\to\infty$ $n\to\infty$

A similar result holds for $\{gy_n\}$. Standard arguments in IFMS then show $\{gx_n\}$ and $\{gy_n\}$ are Cauchy sequences.

Step 3: Convergence to Best Proximity Point. By completeness, there exist $p, q \in A$ such that $gx_n \to p$ and $gy_n \to q$. Using (C3) (G-continuity or G-regularity), we pass to the limit in the iterative equations. From the definition of $A_0(t)$ and the construction, we deduce:

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$M(gp, F(p,q), t) = M(A,B,t)$ and $N(gp, F(p,q), t) = N(A,B,t)$,
$M(gq, F(q,p), t) = M(A,B,t)$ and $N(gq, F(q,p), t) = N(A,B,t)$.
Thus, (p, q) is the desired coupled coincidence best proximity point.

Theorem 3.3 (Uniqueness). *If, in addition to the conditions of Theorem 3.2, for every* $(x,y), (x^*,y^*) \in A \times A$*, there exists* $(u,v) \in A \times A$ *that is comparable to both (with respect to the graph and the partial order), then the coupled coincidence best proximity point is unique.*

Proof. Suppose (p, q) and (p^*, q^*) are two coupled coincidence best proximity points. By the comparability condition, there exists $(u, v) \in A \times A$ comparable to both. Construct se- quences as in Theorem 3.2 starting from (u, v). Using the contraction condition and prop- erties of φ, we can show that $M(gp, gp^*, t) = 1$ and $N(gp, gp^*, t) = 0$ for all $t > 0$, implying $gp = gp^*$. Similarly, $gq = gq^*$. Thus, the point is unique.

4. Coupled Fixed Point Theorem:

If $A = B = X$ in Theorem 3.2, then dist$(A, B) = 0$ in the metric sense, which corresponds to $M(A, B, t) = 1$ and $N(A, B, t) = 0$ for all $t > 0$ in the IFMS context. A coupled coincidence best proximity point then becomes a coupled coincidence fixed point: $gp = F(p, q)$ and $gq = F(q, p)$.

Corollary 4.1. *Let* $(X, M, N, *, \diamond, G, \preceq)$ *be a complete IFMS endowed with a graph and a partial order. Let* $F : X \times X \to X$ *and* $g : X \to X$ *be surjective such that:*

- *F has the mixed* $(G, \preceq)$*-monotone property.*
- *There exist* $x_0, y_0 \in X$ *with* $(gx_0, F(x_0,y_0)), (gy_0, F(y_0,x_0)) \in E(G)$*, and* $gx_0 \preceq F(x_0,y_0)$, $gy_0 \succeq F(y_0,x_0)$.
- *F is a* $(G,\preceq,\varphi)$*-contraction (with* $A = B = X$*).*
- *F is* $(G,\preceq)$*-continuous, or* (X,M,N,G) *is G-regular.*

Then, F and g have a coupled coincidence fixed point. If comparability holds for all pairs, it is unique.

Remark 4.2. *By choosing g = I (identity),* $E(G) = X \times X$ *(complete graph), and* $\preceq$ *as equal- ity, Corollary 4.1 reduces to the classical coupled fixed point theorem of Bhaskar and Lakshmikantham [4] in the context of IFMS.*

5. Applications and Examples:

Example 5.1. *Let* $X = [0,2] \times \mathbb{R}$*. Define for* $(x_1,y_1),(x_2,y_2) \in X$*:*

$$M((x_1,y_1),(x_2,y_2),t) = \frac{t}{t + |x_1 - x_2| + |y_1 - y_2|},$$

$$N((x_1,y_1),(x_2,y_2),t) = \frac{|x1 - x2| + |y1 - y2|}{t + |x_1 - x_2| + |y_1 - y_2|},$$

with $a * b = ab$, $a \diamond b = \min(1, a+b)$*. Then* $(X, M, N, *, \diamond)$ *is a complete IFMS. Let* $A = \{(0,x) : 0 \le x \le 1\}$, $B = \{(2,y) : 0 \le y \le 1\}$*. Then*

$$M(A,B,t) = \frac{t}{t+2}, \quad N(A,B,t) = \frac{2}{t+2}.$$

Define the graph G by: $((a,b),(c,d)) \in E(G)$ *if and only if* $a, c \in \{0, 2\}$ *and* $b \le d$*. Define partial order:* $(a,b) \preceq (c,d)$ *if and only if* $a = c$ *and* $b \le d$*.*
Define $g : A \to A$ *by* $g(0,x) = (0,x/2)$ *and* $F : A \times A \to B$ *by* $F((0,x),(0,y)) = (2, \frac{x+y}{4})$*. Let* $\varphi(s,t) = \min(s,t)$.
One can verify all conditions of Theorem 3.2. The point $(p,q) = ((0,0),(0,0))$ *satisfies* $g(0,0) = (0,0)$ *and*

$$M(gp,F(p,q),t) = M((0,0),(2,0),t) = \frac{t}{t+2} = M(A,B,t),$$

$$N(gp,F(p,q),t) = \frac{2}{t+2} = N(A,B,t).$$

Hence, it is a coupled coincidence best proximity point.

Application to Nonlinear Integral Equations:

Consider the system of integral equations:

$$u(t) = \int_0^1 K_1(t, s, u(s), v(s))ds + f(t), \tag{1}$$

$$v(t) = \int_0^1 K_2(t, s, v(s), u(s))ds + f(t), \tag{2}$$

for $t \in [0, 1]$, where $f \in L^\infty([0, 1])$, and K_1, K_2 are kernels such that the mappings are mixed monotone. Let $X = L^\infty([0, 1]) \times L^\infty([0, 1])$. We can define an IFMS on X using the es- sential supremum norm. Define a graph G where $((u_1, v_1), (u_2, v_2)) \in E(G)$ if $u_1(t) \leq u_2(t)$ and $v_1(t) \geq v_2(t)$ a.e., and a partial order similarly. Under Lipschitz conditions on K_1, K_2 that translate into a $(G, \preceq, \varphi)$-contraction, Theorem 3.2 (in its coupled fixed point version, Corollary 4.1) guarantees the existence of a solution $(u, v) \in X$ to the system.

6. Conclusion and Future Work:

This paper has successfully established coupled coincidence best proximity point theorems in intuitionistic fuzzy metric spaces simultaneously equipped with a directed graph and a partial ordering. The results significantly generalize and unify various strands of fixed point theory, offering a versatile tool for problems involving uncertainty, relational constraints, and ordered structures.

Future Research Directions:

- Extending these theorems to **multivalued mappings**.
- Investigating **common coupled best proximity points** for pairs of mappings.
- Replacing the t-norm/t-conorm with more general operators.
- Exploring concrete applications in **fuzzy optimization, decision theory, and im- age processing** where the graph represents spatial or functional relationships and intuitionistic fuzzy metrics handle noise/uncertainty.
- Developing **algorithmic implementations** for approximating such points based on the iterative sequences constructed in the proofs.

References:

1. Alaca, C., Turkoglu, D., & Yildiz, C. (2006). Fixed points in intuition- istic fuzzy metric spaces. *Chaos, Solitons & Fractals*, 29(5), 1073-1078. https://doi.org/10.1016/j.chaos.2005.08.066
2. Alghamdi, M. A., Berzig, M., & Karapınar, E. (2013). Best proximity point results in generalized metric spaces. *Abstract and Applied Analysis*, 2013. https://doi.org/10.1155/2013/832796
3. Banach, S. (1922). Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales. *Fundamenta Mathematicae*, 3, 133-181.
4. Bhaskar, T. G., & Lakshmikantham, V. (2006). Fixed point theorems in partially or- dered metric spaces and applications. *Nonlinear Analysis: Theory, Methods & Appli- cations*, 65(7), 1379-1393. https://doi.org/10.1016/j.na.2005.10.017
5. Choudhury, B. S., Kundu, A., & postolache, M. (2011). A coupled fixed

point result in partially ordered metric spaces. *Demonstratio Mathematica*, 44(3), 495-506.

6. Grabiec, M. (1988). Fixed points in fuzzy metric spaces. *Fuzzy Sets and Systems*, 27(3), 385-389. https://doi.org/10.1016/0165-0114(88)90064-4
7. Guo, D., & Lakshmikantham, V. (1987). Coupled fixed points of nonlinear operators with applications. *Nonlinear Analysis: Theory, Methods & Applications*, 11(5), 623- 632. https://doi.org/10.1016/0362-546X(87)90077-0
8. Jachymski, J. (2008). The contraction principle for mappings on a metric space with a graph. *Proceedings of the American Mathematical Society*, 136(4), 1359-1373. https://doi.org/10.1090/S0002-9939-07-09110-1
9. Kramosil, I., & Michalek, J. (1975). Fuzzy metrics and statistical metric spaces. *Ky- bernetika*, 11(5), 336-344.
10. Nieto, J. J., & Rodr´ıguez-López, R. (2005). Contractive mapping theorems in partially ordered sets and applications to ordinary differential equations. *Order*, 22(3), 223-
11. 239. https://doi.org/10.1007/s11083-005-9018-5
12. Park, J. H. (2004). Intuitionistic fuzzy metric spaces. *Chaos, Solitons & Fractals*, 22(5), 1039-1046. https://doi.org/10.1016/j.chaos.2004.02.051
13. Sankar Raj, V. (2011). A best proximity point theorem for weakly contractive non-self- mappings. *Nonlinear Analysis: Theory, Methods & Applications*, 74(14), 4804-4808. https://doi.org/10.1016/j.na.2011.04.052
14. Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X
15. Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1), 87-96. https://doi.org/10.1016/S0165-0114(86)80034-3
16. Schweizer, B., & Sklar, A. (1960). Statistical metric spaces. *Pacific Journal of Mathe- matics*, 10(1), 313-334.
17. George, A., & Veeramani, P. (1994). On some results in fuzzy metric spaces. *Fuzzy Sets and Systems*, 64(3), 395-399. https://doi.org/10.1016/0165-0114(94)90162-7
18. Sintunavarat, W., & Kumam, P. (2012). Coupled best proximity point the- orem in metric spaces. *Fixed Point Theory and Applications*, 2012(1), 93. https://doi.org/10.1186/1687-1812-2012-93
19. Samet, B., & Vetro, C. (2010). Coupled fixed point theorems for multi-valued nonlinear contraction mappings in partially ordered metric spaces. *Nonlinear Analysis: Theory, Methods & Applications*, 74(12), 4260-4268. https://doi.org/10.1016/j.na.2011.04.007
20. Agarwal, R. P., El-Gebeily, M. A., & O'Regan, D. (2008). Generalized contrac- tions in partially ordered metric spaces. *Applicable Analysis*, 87(1), 109-116. https://doi.org/10.1080/00036810701556151

21. Berinde, V. (2012). Coupled fixed point theorems for -contractive mixed monotone mappings in partially ordered metric spaces. *Nonlinear Analysis: Theory, Methods & Applications*, 75(6), 3218-3228. https://doi.org/10.1016/j.na.2011.12.021
22. Lakshmikantham, V., & Ć irić, L. (2009). Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces. *Nonlinear Analysis: Theory, Methods & Applications*, 70(12), 4341-4349. https://doi.org/10.1016/j.na.2008.09.020
23. Kirk, W. A., Reich, S., & Veeramani, P. (2003). Proximinal retracts and best proximity pair theorems. *Numerical Functional Analysis and Optimization*, 24(7-8), 851-862. https://doi.org/10.1081/NFA-120026380
24. Eldred, A. A., & Veeramani, P. (2006). Existence and convergence of best proxim- ity points. *Journal of Mathematical Analysis and Applications*, 323(2), 1001-1006. https://doi.org/10.1016/j.jmaa.2005.10.081
25. Mongkolkeha, C., & Kumam, P. (2013). Best proximity points for asymptotic proximal pointwise contraction mappings. *Abstract and Applied Analysis*, 2013. https://doi.org/10.1155/2013/329837
26. Basha, S. S. (2011). Best proximity point theorems on partially ordered sets. *Opti- mization Letters*, 7(5), 1035-1043. https://doi.org/10.1007/s11590-012-0476-6
27. Beg, I., Butt, A. R., & Radojević, S. (2010). The contraction principle for set valued mappings on a metric space with a graph. *Computers & Mathematics with Applica- tions*, 60(5), 1214-1219. https://doi.org/10.1016/j.camwa.2010.06.003
28. Bojor, F. (2010). Fixed point theorems for Reich type contractions on metric spaces with a graph. *Nonlinear Analysis: Theory, Methods & Applications*, 72(5), 2319-2328.
29. Abbas, M., & Jungck, G. (2008). Common fixed point results for noncommuting map- pings without continuity in cone metric spaces. *Journal of Mathematical Analysis and Applications*, 341(1), 416-420. https://doi.org/10.1016/j.jmaa.2007.09.070
30. Mustafa, Z., & Sims, B. (2006). A new approach to generalized metric spaces. *Journal of Nonlinear and Convex Analysis*, 7(2), 289-297.
31. Ran, A. C., & Reurings, M. C. (2004). A fixed point theorem in partially ordered sets and some applications to matrix equations. *Proceedings of the American Mathemat- ical Society*, 132(5), 1435-1443. https://doi.org/10.1090/S0002-9939-03-07220-4
32. Suzuki, T. (2008). A generalized Banach contraction principle that characterizes met- ric completeness. *Proceedings of the American Mathematical Society*, 136(5), 1861-1869. https://doi.org/10.1090/S0002-9939-07-09055-7