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Original Article

ANALYSIS OF BLACK HOLE THERMODYNAMICS UNDER GENERALIZED UNCERTAINTY PRINCIPLE FROM THE DOUBLY SPECIAL RELATIVITY

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Abstract:

In this paper, we investigate effect of the generalized uncertainty principle (GUP) under the doubly special relativity (DSR) on the thermodynamics properties of the topological charged black hole in Anti-de Sitter (AdS) space only in the spherical horizon case have. Our study is based on a heuristic analysis of a particle which is captured by the black hole. We also report here some results of a usual analytical computation. However, we obtain the black hole thermodynamic properties as familiar concepts such as temperature, entropy and heat capacity under DSR-GUP. we also compare our both analytical results. Beside we discuss the behavior of the corrected thermodynamic properties vs. changes of black hole characteristics under different condition.

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Introduction:

In quantization of gravity, the concept of minimum measurable length is included when considering the discreteness of the space time which occurs beyond the Planck energy scale refs. [1–6]. By considering ~ = 1, the ordinary Heisenberg uncertainty principle is given by ΔxΔp > 1 2 that it does not include minimum measurable length because in the high momentum limit, Δx goes to zero. Therefore, deforming the ordinary Heisenberg uncertainty is a clear way to import the concept of minimum measurable length into quantum mechanics. Thus one can improve the ordinary Heisenberg uncertainty principle which is called Generalized uncertainty principle (GUP). This principle will lead to the corrected the thermodynamic quantities refs. [7–28]. These quantities have been successfully considered to describe and investigate behavior of black holes. Recently, by embracing two invariant fundamental scales, the speed of light and an energy scale distinguished with planck energy, the deformation of the special relativity was considered ref. [6]. Because this theory has two invariant scale, this modified form of theory is usually called a doubly special relativity (DSR). In the last decade, the new form of GUP based on DSR was introduced and in this manuscript, we propose a new form of GUP as follows

$$[\hat{x}, \hat{p}] = i\left(1 - \frac{|\hat{p}|}{\kappa}\right)^2, \tag{1.1}$$

This gives

$$\Delta\hat{x}\Delta\hat{p} \geqslant \frac{1}{2}\left[1 - 2\frac{\langle|\hat{p}|\rangle}{\kappa} + \frac{1}{\kappa^2}(\Delta\hat{p})^2\right] \tag{1.2}$$

Where from now on we call the relation (1.2) DSR-GUP. The expectation value of in Eq. (1.2) deepens on the wave function and this value runs from 0 to $\frac{\kappa}{2}$ ref. [6]. Thus (1.2) can

2 A class of static and spherically black holes

In the present paper, we would like to investigate the topological black holes in the four-dimensional space-time. We consider a static and spherical black hole black hole solution as follows refs. [9–11]

$$dS^2 = -F(r)dt^2 + F^{-1}(r)dr^2 + r^2 d\Omega^2, \tag{2.1}$$

where r is cosmological radius and $d\Omega^2$ is the line element of a two-dimensional hypersurface Ω with constant curvature $2K$

$$d\Omega^2 = \begin{cases} d\theta^2 + \sin^2\theta\, d\phi^2 & \text{for } K = 1 \\ d\theta^2 + \theta^2\, d\phi^2 & \text{for } K = 0 \\ d\theta^2 + \sinh^2\theta\, d\phi^2 & \text{for } K = -1 \end{cases} \tag{2.2}$$

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and the metric function $F(r)$ define as

$$F(r) = K - \frac{8\pi GM}{\Sigma_K r} + \frac{16\pi^2 G^2 Q^2}{\Sigma_K^2 r^2} + \frac{r^2}{\ell^2}, \tag{2.3}$$

where M, Σ_K and Q have identified mass, volume and electric charge and ℓ is cosmological radius and $\Lambda = -3\ell^{-2}$ is the negative cosmological constant. Here for simplicity, we have considered $\frac{4\pi G}{\Sigma_K} = 1$. Without loss of generality, we can consider $K = 1$ for spherical horizon, $K = 0$ for planar/toroidal horizon and $K = -1$ for hyperbolic horizon.

According to the metric function in (4.1) with considering $K = 1$ and $F(r_0) = 0$, the black hole mass is

$$M = \frac{r_0}{2} + \frac{Q^2}{2r_0} - \frac{\Lambda}{6} r_0^3, \tag{2.4}$$

Where r_0 denotes the position of the event horizon of the black hole. By defining the thermodynamic pressure as $P = -\Lambda/8\pi$, Then the surface gravity of the black hole is obtained as follows ref. [21]

$$\kappa = \frac{F'(r_0)}{2} = \frac{-Q^2 + r_0^2 + 8\pi P r_0^4}{2r_0^3}, \tag{2.5}$$

In this research, we study the spherical horizon case and in this case, the solution (2.1) is the Reissner-Nordstrom-anti-de Sitter black hole spacetime and the event horizon has the topology S^2.

In the semi-classical case, the temperature and entropy for the black hole are

$$S_0 = \frac{A}{4\hbar}, \qquad T_0 = \frac{\kappa\hbar}{2\pi}. \tag{3.4}$$

When the particle absorbs by black hole, the smallest increase in the area of a black hole can be considered as ref. [28]

$$A \sim Xm, \tag{3.5}$$

by defining X and m as the particle's size and mass, respectively and By considering $(\Delta S)_{\text{min}} = \text{Ln}2$. ref. [28] and by Knowing that in quantum mechanics, the momentum uncertainty is not allowed to be greater than the mass ($\Delta p \leqslant m$), we have studied a particle by a wave packet that the width of wave packet is decribed as the standard deviation of X distribution i.e. the position uncertainty, which can be identified as the characteristic size of the particle ($X \sim \Delta x$). Thus the representation (3.5) can be rewritten as follows

$$A \sim Xm \geqslant \Delta x \Delta p. \tag{3.6}$$

When a particle is captured by black hole, Δx should not be greater than a specific scale which minimizes ΔA. This characteristic size should be related to the black hole, if ΔA_{min} is expected to describe an intrinsic property of the horizon.

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Conclusions:

In this manuscript, by considering the black hole with the special topology (K = 1), we have obtained the temperature of the charged AdS black hole under DSR-GUP. One can see obtained results are different of their semiclassical form. Also the corrected entropy has been found for the uncharged black hole and as is shown in figures corresponding to of the corrected forms of entropy, temperature and heat capacity, there is the unique physical critical point. Our numerical data describe the thermodynamic properties in detail under the position of the event horizon of the black hole r0. As seen in numerical results, DSR-GUP influences on the temperature, the entropy and the heat capacity. The results of our work are comparable with the previous works of other authors in the literature for generalized uncertainty.

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