



TURBULENT MIXING AND TRANSITION CRITERIA OF FLOWS INDUCED BY HYDRODYNAMIC INSTABILITIES

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ABSTRACT:

In diverse areas of science and technology, including inertial confinement fusion (ICF), astrophysics, geophysics, and engineering processes, turbulent mixing induced by hydrodynamic instabilities is of scientific interest as well as practical significance. Because of the fundamental roles they often play in ICF and other applications, three classes of hydrodynamic instability-induced turbulent flows—those arising from the Rayleigh-Taylor, Richtmyer-Meshkov, and Kelvin-Helmholtz instabilities—have attracted much attention. ICF implosions, supernova explosions, and other applications illustrate that these phases of instability growth do not occur in isolation but instead are connected so that growth in one phase feeds through to initiate growth in a later phase. Essentially, a description of these flows must encompass both the temporal and spatial evolution of the flows from their inception. Hydrodynamic instability will usually start from potentially infinitesimal spatial perturbations, will eventually transition to a turbulent flow, and then will reach a final state of a true multiscale problem. Indeed, this change in the spatial scales can be vast, with hydrodynamic instability evolving from just a few microns to thousands of kilometers in geophysical or astrophysical problems. These instabilities will evolve through different stages before transitioning to turbulence, experiencing linear, weakly, and highly nonlinear states. The challenges confronted by researchers are enormous. The inherent difficulties include characterizing the initial conditions of such flows and accurately predicting the transitional flows. Of course, fully developed turbulence, a focus of many studies because of its major impact on the mixing process, is a notoriously difficult problem. In this pedagogical review, we will survey challenges and progress and also discuss outstanding issues and future directions.

Keywords: Hydrodynamic Instabilities, Landau Model, Bifurcation, Rayleigh-Bénard Instability, Bénard-Marangoni Instability, Taylor-Couette Instability, Kelvin-Helmholtz Instability, Subcritical Instabilities.

INTRODUCTION:

Transition from laminar to turbulent flows has generally been studied by considering the linear and weakly nonlinear evolution of small disturbances to the laminar flow. That approach has been fruitless for many shear flows, and the last hope for its success has been the existence of transient growth phenomenon. The physical origin of those linear transient effects is elucidated, revealing serious limitations both of previous analyses and of the phenomena themselves, which preclude them from causing direct transition. Nonetheless, some transient effects are symptomatic of one element of a nonlinear process that becomes self-sustaining at a small enough dissipation. The process is identified, and its description requires a departure from the traditional focus on the laminar flow. A theory is outlined in which the mean flow has an intrinsic spanwise variation. Evidence indicates this is also the central mechanism in the near wall-region of fully turbulent shear flows.

METHODOLOGY:

The instability of fluid flows is a key topic in classical fluid mechanics because it has huge repercussions for applied disciplines such as chemical engineering, hydraulics, aeronautics, and geophysics. As a decade's experience of teaching post graduate students in fluid dynamics, this book brings the subject to life by emphasizing the physical mechanisms involved. The theory of dynamical systems provides the basic structure of the exposition, together with asymptotic methods. Wherever possible, Charru discusses the phenomena in terms of characteristic scales and dimensional analysis. The book includes numerous experimental studies, with references to videos and multimedia material, as well as over 150 exercises which introduce the reader to new problems. In the first part, which is more mathematical than physical, we define the fundamental ideas. These ideas are then illustrated by examples borrowed from hydrodynamics and the physics of liquids. We close the chapter with a brief presentation of the idea of transient growth, which is related to nonorthogonality of the eigenvectors of a linear system. The time evolution of a discrete (noncontinuous) physical system is generally governed by differential

equations following physical conservation principles and the laws describing the phenomenologically. These equations can often be written as a system of first-order ordinary differential equations (ODEs) of the form (see, e.g., Glendinning (1994)): $\dot{x}_i = X_i(x_1, \dots, x_n, t)$, $i = 1, \dots, n$. (1.1) The remainder of this chapter will consider only autonomous systems, in which time does not appear explicitly on the right-hand side. The variables x_i are called the degrees of freedom of the system.¹ As an example, let us consider a simple damp nonlinear pendulum whose vertical position is specified by the angle θ . Its equation of motion $\frac{d^2\theta}{dt^2} + \mu \frac{d\theta}{dt} + \omega^2 \sin\theta = 0$ (1.2).

CONCLUSION:

Any solution of a system of ODEs for a given initial condition can be represented by a curve in the space of the degrees of freedom, called the phase space. For the system (1.3) the phase space is the (x_1, x_2) plane. Figure 1.1 shows typical trajectories corresponding to given initial conditions for $\mu=0$ and $\mu>0$. The case $\mu=0$ corresponds to a no dissipative oscillator (i.e., where the mechanical energy remains constant), and the case $\mu>0$ corresponds to a dissipative oscillator (where the mechanical energy decreases over time). A representation of this type which depicts the essential features of the solutions of a system of ODEs is called the phase portrait, which allows the trajectory to be plotted qualitatively for any given initial condition.

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portrait of the system. The phase portrait can be guessed easily for a system as elementary as the pen durum. For more complicated systems the first step is to determine the fixed points and study their stability. When there are several fixed points the second important step is to determine which fixed point the system evolves for initial conditions. The ensemble of initial conditions resulting in motion to a particular fixed point is called the basin of attraction of that fixed point. Stability of a fixed-point Fixed points the equilibrium states of a physical system correspond to the stationary.

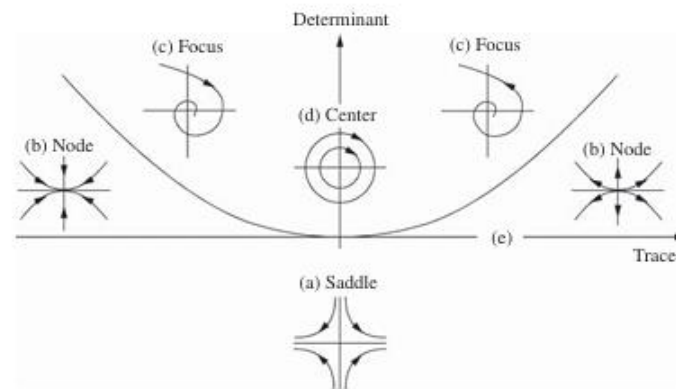


Figure 1.2 Types of fixed point in $\mathbf{R}^2$. The parabola corresponds to $\text{tr}^2 \mathbf{L} - 4\det \mathbf{L} = 0$ (discriminant of the characteristic polynomial equal to zero).

Special situation occurs when all the eigenvalues have negative real part except for one (or several) which have zero real part. The fixed point is then non-hyperbolic, and we can learn nothing about its stability from the linear stability analysis. Its stability is therefore determined by the nonlinear terms, whose effect can be stabilizing or destabilizing. Let us take as an example the oscillator described by the system (1.3) in the non-dissipative case ($\mu=0$) with an additional force $\beta(d\theta/dt)^3$. The system linearized about the fixed point $(0,0)$ possesses two purely imaginary eigenvalues $\pm i\omega_0$, and so the linear stability analysis tells us nothing. However, in this particular case it can be shown simply, without linearization, that the fixed point is stable for $\beta>0$ and unstable for $\beta<0$ and increasing for β . Saddle-node bifurcation 7 Let us consider the mechanical system represented in Figure. An arm of length l is attached to a pivot at its lower end and holds a mass m at its other end; its angular position is given by the angle θ . One end of a helical torsion spring with spring constant C is attached to the arm, while the other end of the spring is attached to a plane

inclined at an angle α with respect to the horizontal. The spring tends to restore the arm to the direction perpendicular to the attached plane. We also include a moment of viscous friction $-mgl\tau^*d\theta/dt$ about the pivot, where τ^* is a relaxation time. Denoting the mass, length, and time scales as m , l , and $\sqrt{l/g}$, the oscillator potential energy in the gravitational field can be written as $V(\omega^2, \alpha, \theta) = \frac{\omega^2}{2} (\theta - \alpha)^2 + \cos\theta - 1$, where the characteristic frequency ω is defined as $\omega^2 = C mgl$. (1.8) (1.9) In terms of these scales the friction moment takes the form $-\tau d\theta/dt$, where $\tau = \tau^*/\sqrt{l/g}$ is the dimensionless relaxation time. The equation of motion is then $d^2\theta/dt^2 + \tau d\theta/dt = -\partial V/\partial\theta$. (1.10) This equation can be rewritten as a dynamical system of two ODEs in the phase space $(\theta, d\theta/dt)$. The fixed points (equilibrium states) are defined by $d\theta/dt = 0$, and θ is the root of the equation for the potential extrema: $0 = \partial V/\partial\theta = \omega^2(\theta - \alpha) - \sin\theta$.

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Introduction

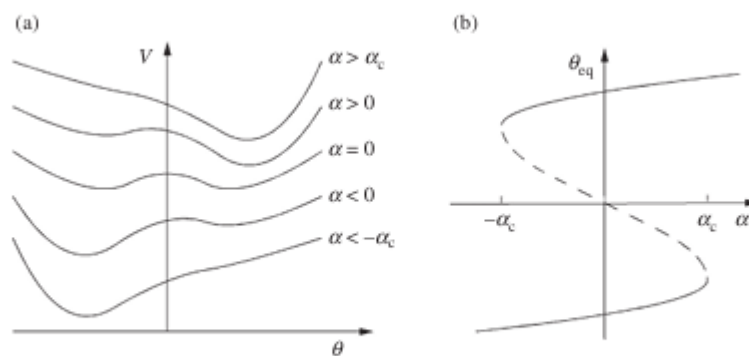


Figure 1.4 (a) The potential $V(\theta)$ for various inclinations α (the relative vertical positions of the various curves are arbitrary). (b) The bifurcation diagram: (—) stable states, (---) unstable states.

The dependence of the equilibrium states on the two parameters ω^2 and α can be determined graphically or, for $|\alpha|$ small and ω^2 near unity, by a Taylor series expansion about $\theta = 0$. For $\alpha = 0$ the potentials at the equilibrium points θ_- and θ_+ are the same (Figure 1.4a). For $|\alpha|$ small and $\omega^2 \gg 0$ (whose corresponding fixed points are nodes). For $\alpha < 0$ is only metastable. The situation is reversed for $\alpha > 0$. Let us consider the system in the state θ_- with α positive and small (Figure 1.4b). As α increases, the metastable equilibrium state θ_- and the unstable equilibrium state θ_0 approach each other, and there exists a critical inclination α_c for which the two equilibrium states merge. For $\alpha > \alpha_c$, the system jumps to the stable branch θ_+ . For $\alpha = \alpha_c$, the phase portrait of the system therefore undergoes a qualitative change when the stable node $(\theta_-, 0)$ and the unstable

saddle $(0, 0)$ coalesce. This qualitative change corresponds to a bifurcation: for $\alpha = \alpha_c$, an eigenvalue of the system linearized about each of the fixed points $(0, 0)$ and $(0, -1)$ crosses the imaginary axis (the proof is left as an exercise). The corresponding bifurcation is called a saddle-node bifurcation. A similar bifurcation occurs for decreasing α when α reaches the value $-\alpha_c$. Figure 1.4b, which shows the fixed points as a function of the parameter α , is called the bifurcation diagram. At each bifurcation the system jumps from one branch to another, and the critical value of the bifurcation parameter α is depending on whether it is increasing or decreasing: the system displays hysteresis.

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