



Quantum Turbulent Flows: A Model for Classical Turbulence

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Abstract:

A review of quantum turbulence, that is, the turbulent motion of quantized vortex lines in superfluid helium. Our discussion concentrates on the turbulence produced by steady, uniform heat flow in a pipe, but touches on other turbulent flows as well. We have attempted to motivate the study of quantum turbulence and discuss briefly its connection with classical turbulence. We include background on the two-fluid model and mutual friction theory, examples of modern experimental techniques, and a brief survey of phenomenology.

Keywords: *Quantum Turbulent Flows, Model for Classical Turbulence*

Introduction:

There are two well-established techniques for the analysis of hydrodynamic stability/instability, namely, spectral methods (normal modes) (see, for example, Chandrasekhar [3], Drazin and Reid [3]) and energy methods (see, for example, Arnold [4], Holm et al. [2], Arnold and Khesin [1], Vladimirov [1]). Recently, a modification of the spectral method called the pseudospectral method was proposed by Trefethen [2], Trefethen et al. [2], Waleffe [1] and others. These conventional techniques proved to be very useful for studying the stability/instability of some steady flows with relatively simple structure but their applicability to more complicated flows has proved to be limited. In this paper we emphasize a third technique, namely, the geometrical optics method which is capable of probing the instability of general classes of three-

dimensional inviscid flows. (By its very nature this method cannot prove stability.) In contrast to spectral and energy methods, the geometrical optics method is specifically designed for studying highly localized short-wave perturbations of an arbitrary background flow. These perturbations are localized wave envelopes moving along the trajectories of fluid elements. The evolution of a particular envelope is governed by a characteristic system of ordinary equations along the relevant trajectory. In the language of geometrical optics the characteristic equations consist of the eikon for the wave vector and the transport equation for the velocity amplitude.

Discussion:

Turbulence is ubiquitous in the universe. It occurs in systems ranging in size from tens of micro-meters, such as cryogenic helium channels and clouds of

few thousand atoms confined by optical lasers, to planetary scales, such as large-scale atmospheric and oceanic flows, to astrophysical scales characterizing nebulae of interstellar gas. The dynamics of turbulent flows is shaped by the mutual interplay of nonlinearly interacting vortical structures spanning a wide range of scales. Properties of these vortices may differ significantly depending on the nature of the fluid. In classical viscous fluids, vortices are unconstrained: classical vortices, such as a teaspoon-stirred vortex in a cup of coffee or an atmospheric tornado, may possess any shape, orientation, size and strength, the latter quantified by the circulation of the fluid's velocity field around a closed loop surrounding the vortex structure. Very dissimilar properties instead characterize vortices in quantum fluids. The latter are fluids which, due to Bose–Einstein condensation of the constituent particles, exhibit quantum-mechanical properties at a macroscopic level. Examples of such fluids are liquid phases of He 3 and He 4 (Barenghi, Skrbek & Sreenivasan Reference Barenghi, Skrbek and Sreenivasan2023), ultracold atomic gases (White, Anderson & Bagnato Reference White, Anderson and Bagnato2014), polariton condensates (Panico et al. Reference Panico, Comaron, Matuszewski, Lanotte, Trypogeorgos, Gigli, Giorgi, Ardizzone, Sanvitto and Ballarini2023) and the interior of neutron stars (Haskell & Melatos Reference Haskell and Melatos2015). In the present work we will mainly focus on the low temperature liquid phase of He 4 ,

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commonly referred to as helium II, which can be described as an intimate mixture of two inseparable fluid components (Landau Reference Landau1941): the superfluid component, related to the Bose–Einstein condensate, possessing vanishing entropy and viscosity, hence similar to an inviscid, classical Euler fluid; the normal fluid, almost akin to a classical viscous fluid, carrying the whole entropy of helium II. Two density and velocity fields characterise helium II, where two motions can therefore occur simultaneously at the same position in space. Whereas the total density is almost constant in the temperature range where the helium II phase exists, $0 \leq T \leq T_\lambda = 2.17 \text{ K}$ (T_λ being the transition temperature), helium II is almost entirely superfluid below 1.3 K and fully normal at T_λ . Above T_λ , the liquid phase of He 4 is a classical Newtonian fluid. Most importantly, quantum-mechanical restrictions confine the whole vorticity of the superfluid to effectively one-dimensional vortex filaments of quantised circulation $\kappa = h/m$, where h is the Planck constant and m the mass of a helium atom. These quantum vortices interact collectively with each other via the Biot–Savart law, may reconnect (Bewley et al. Reference Bewley, Paoletti, Sreenivasan and Lathrop2008; Serafini et al. Reference Serafini, Galantucci, Iseni, Bienaimé, Bisset, Barenghi, Dalfovo, Lamporesi and Ferrari2017) as a result of small-scale quantum compressible effects at the vortex core level and exert a so-called mutual friction dissipative force on the normal

fluid (Hall & Vinen Reference Hall and Vinen1956). In brief, the motion of superfluid vortices is influenced by the normal fluid whose dynamics, in turn, is also governed by the mutual friction force exerted by the vortices: the overall motion of helium II is the result of two-phase, fully coupled dynamics (Galantucci et al. Reference Galantucci, Baggaley, Barenghi and Krstulovic2020).

Methodology:

Despite the strongly dissimilar nature of classical and quantum fluids just described, under certain conditions, turbulence in such fluids may surprisingly exhibit very similar features. The work by Blaha, Xu & La Mantia (Reference Blaha, Xu and La Mantia2025) precisely reports evidence in this direction, showing that starting vortices shed by airfoils accelerating uniformly from rest behave very similarly in helium II and classical Newtonian fluids. In particular, the trajectories of quantum vortices determined experimentally are almost indistinguishable from the trajectories of vortices shed in a classical fluid computed via a direct numerical simulation of the Navier–Stokes equations. In addition, a recent study by the same group (Švančara & La Mantia Reference Švančara and La Mantia2022) shows that macroscopic vortex rings in helium II propagate in a similar manner to turbulent vortex rings in a Newtonian fluid, as confirmed numerically (Galantucci et al. Reference Galantucci, Rickinson, Baggaley, Parker and Barenghi2023a). The significant relevance of the investigation by *Sanjay Kumar Shailendra*

Blaha et al. (Reference Blaha, Xu and La Mantia2025) is hence its contribution in constructing a detailed map identifying the parameter range (temperatures, flow length scales, flow generation methods) where quantum turbulence (QT) in helium II behaves classically. The construction of this map is of fundamental importance not only to enhance our understanding of why turbulence in quantum fluids shows similarities to turbulence in classical viscous fluids. The extremely small kinematic viscosity of helium II (up to three orders of magnitude smaller than that of air) would indeed also allow the investigation of high Reynolds number flows of classical Newtonian fluids in helium II experimental facilities at significantly smaller scales than found in typical wind tunnels. Finally, as we shall briefly describe in the next section, the similarities between QT and classical turbulence (CT), strongly support the conjecture that QT can be considered the skeleton of classical flows: the study of the dynamics of a collection of discrete filaments of fixed circulation could enhance our understanding of the still open issues in CT.

Discussion: Quasi-Classical Quantum Turbulence

The characteristic presence of a tangle of quantised vortices in turbulent flows of helium II immediately introduces an additional length scale to helium II systems, the average spacing between vortices $\ell = 1/\sqrt{L}$, where L is the vortex line density defined as the average vortex

length per unit volume. The current understanding is that, when turbulence is driven mechanically at scales larger than the average intervortex spacing ℓ , three-dimensional QT exhibits a classical behaviour in terms of energy decay, the energy spectrum, the statistical distribution of velocity and acceleration components and circulation statistics.

In QT, the decay of the turbulent kinetic $E_{[f_0]}(t)$ is investigated in terms of the decay of the vortex density $L_{[f_0]}(t)$ assuming, as in a rotating bucket (Peretti et al. Reference Peretti, Vessaire, Durozoy and Gibert2023), that the superfluid vorticity ω_s is related to L by the expression $\omega_s = \kappa \times L$. In this framework, the classical energy decay rate $E_{[f_0]}(t) \sim t^{-2}$ (Touil, Bertoglio & Shao Reference Touil, Bertoglio and Shao2002) maps in QT to $L_{[f_0]}(t) \sim t^{-3/2}$. Measurements in helium II indeed observe this vortex line density decay rate, in distinct experimental settings (Skrbek & Sreenivasan Reference Skrbek and Sreenivasan2012): towed-grid experiments, impulsive spin down of cubic shaped containers and intense ion injection.

The other remarkable similarity between mechanically driven QT at large scales concerns the energy spectrum $E_{[f_0]}(k)$, where k is the wavenumber magnitude. Several experiments have in fact reported that, in the inertial range $kI < k < k\ell$, where kI is the inverse of the energy injection scale and $k\ell = 2\pi/\ell$, the kinetic energy spectrum of helium II displays the

celebrated Kolmogorov Reference Kolmogorov1941) spectrum $E_{[f_0]}(k) \sim k^{-5/3}$. Direct evidence of this classical behaviour has been provided by stirring helium II with counter-rotating discs (Maurer & Tabeling Reference Maurer and Tabeling1998) and by driving helium II along wind tunnels performing measurements behind a grid or a disc (Salort et al. Reference Salort2010). These experimental observations cover a wide temperature range spanning from the critical temperature T_λ down to temperatures where helium II is almost entirely superfluid: all measured spectra perfectly match the Kolmogorov spectrum in the inertial range.

The experimental data have been widely supported by numerical simulations based on distinct models probing the flow at different scales: from the Hall–Vinen–Bekarevic–Khalatnikov model where the smallest resolved scale is larger than ℓ (Roche, Barenghi & Leveque Reference Roche, Barenghi and Leveque2009), to the Lagrangian vortex filament method (VFM) discretising quantum vortices in line elements whose lengths are smaller than ℓ but significantly larger than the vortex core (Baggaley, Laurie & Barenghi Reference Baggaley, Laurie and Barenghi2012), to the Gross–Pitaevskii model, resolving length scales as small as the vortex core (Nore, Abid & Brachet Reference Nore, Abid and Brachet1997). The contribution of numerical simulations has been

fundamental to identifying the mechanisms responsible for the emergence of the Kolmogorov spectrum in QT. The VFM simulations have in fact shown that the Kolmogorov spectrum is directly related to the emergence of quantum vortex bundles, that is regions where quantum vortices are locally aligned and polarised with superfluid vorticity on the lines almost pointing in the same direction.

By coming closer together, these vortex structures can indeed undergo the process of vortex stretching, a mechanism which is known to play an important role in governing the energy cascade (on individual vortices vortex stretching is prevented by the quantisation of the circulation). A similar essential role in shaping the Kolmogorov spectrum is played in CT by elongated regions of large enstrophy (Farge, Pellegrino & Schneider Reference Farge, Pellegrino and Schneider2001). Quantum vortex bundles can hence effectively be described as classical eddies in ordinary turbulence and this explains why in QT classical features emerge at scales larger than ℓ : to observe a classical behaviour, the dynamics needs to be governed by the collective interaction of many quantum vortices.

The role of vortex bundles in QT has been confirmed in a series of recent numerical studies investigating the scaling of the velocity circulation statistics in the $T=0$ limit (Müller et al. Reference Müller, Polanco and Krstulovic2021; Polanco, Müller & Krstulovic Reference Polanco, Müller and Krstulovic2021). By computing the scaling exponents of

circulation moments in the inertial range, in these studies both the self-similar Kolmogorov scalings and intermittency deviations have been observed, the latter closely reflecting the bifractal model for circulation moments recently proposed in CT (Iyer, Sreenivasan & Yeung Reference Iyer, Sreenivasan and Yeung2019). Importantly, the polarisations of vortex bundles are found to determine the Kolmogorov self-similar behaviour, while intermittency corrections arise from the non-trivial spatial distribution of vortices. This unveiled universality of the inertial range dynamics reinforces the view that turbulent flows, both quantum and classical, can be described in terms of the collective, mutually interacting dynamics of discrete and thin vortex filaments of fixed circulation. This universal character of the inertial range dynamics is also supported by the observation of classically shaped probability density functions (PDFs) of velocity and acceleration components in QT at scales larger than ℓ (La Mantia et al. Reference La Mantia, Švančara, Duda and Skrbek2016) employing particle tracking velocimetry of solid hydrogen or deuterium particles (Guo et al. Reference Guo, La Mantia, Lathrop and Van Sciver2014): velocity PDFs coincide with Gaussian distributions, while acceleration distributions follow a classical functional form associated with a log-normal distribution of the acceleration magnitude. When probing scales are lowered below ℓ , quantum effects kick in and the velocity PDFs switch dramatically to power laws arising from the interaction of particles

with individual vortex lines. It is worth mentioning that when the normal fluid is present ($T > 1.3$ K), similarities between CT and QT may also emerge at small scales where the quantum analogue of the classical dissipation anomaly has been observed (Mäkinen et al. Reference Mäkinen, Autti, Heikkinen, Hosio, Hänninen, L'vov, Walmsley, Zavjalov and Eltsov2023; Galantucci et al. Reference Galantucci, Rickinson, Baggaley, Parker and Barenghi2023 b). To conclude this section, it is of essential importance to emphasise that, when QT is generated thermally, e.g. by placing a heater in a closed channel, several non-classical features emerge. For instance, the vortex line density decays as $L_{f0}(t) \sim t^{-1}$ (Vinen Reference Vinen1957) and the energy spectra $E_{f0}(k) \sim k^{-\beta}$ with $\beta > 5/3$ (Gao et al. Reference Gao, Varga, Guo and Vinen2017), evidence of a stronger dissipation arising from the mutual friction force which in thermal QT acts at all scales.

Conclusion:

The recent work by Blaha et al. (Reference Blaha, Xu and La Mantia2025) provides additional evidence that, in flows driven mechanically and at scales larger than the average intervortex spacing, QT displays features akin to CT. This result strongly reinforces the view that the dynamics in the inertial range is universal and, hence, that the backbone of

turbulence, both classical and quantum, is constituted by discrete and thin vortex filaments of fixed circulation. This evocative interpretation hints at the idea that the study of QT could lead to a better understanding of CT. Furthermore, the investigation of Blaha et al. (Reference Blaha, Xu and La Mantia2025) shows that vortex trajectories in QT are very similar to the ones observed in CT, suggesting that quantum-like vortex excitations may be observed on classical vortices.

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