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A REVIEW ON QUANTUM SUBROUTINES AND METHODS OF QUANTUM MACHINE LEARNING WITH SUPERVISED PATTERNS

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Abstract:

As a result of the superiority of Quantum Computing (QC) and the notable success that it has made in a variety of applications, including cryptography, chemistry, Big Data, machine learning, optimization, Internet of Things (IoT), Blockchain, communication, and a great deal more besides. Fully geared toward combining classical machine learning (ML) with quantum information processing (QIP) to build a new field in the quantum world that is called Quantum Machine Learning (QML). The purpose of this new field is to solve and improve problems that were displayed in classical machine learning (e.g., time and energy consumption, kernel estimation). The purpose of this work is to give and synthesise a complete assessment of the recent developments that have been made in the field of quantum machine learning (QML). Specifically, more recent efforts on categorization using QML. In addition, we discuss the findings of over 30 recent articles in the field of Quantum Machine Learning (QML). In this paper, we analyse several encoding strategies for transferring conventional data onto quantum data and suggest a classification strategy for use in the quantum environment. Then, we provide quantum subroutines and a few approaches of quantum computing (QC) as a means of enhancing the functionality of conventional machine learning and accelerating its processing speed (ML). In addition to that, several applications of QML in a variety of domains, as well as difficulties and a vision for the future, will be discussed.

Keywords: Quantum Machine Learning, Quantum Computing, Quantum Bit (Qubit), Quantum Inspired, Hybrid Quantum-Classical, Variational Quantum Classifier, Quantum Classification, Machine Learning.

Introduction:

As is widely known, machine learning (1) plays a key role in data analysis, feature selection, decision-making, pattern categorization, and future predictions for a variety of applications, enabling higher accuracy and efficiency without the need for human judgement. However, with the massive growth in data kinds (such as photographs, text, videos, and recorded audios) and the available computing power, machine learning issues including high-cost learning and kernel estimation emerged. Over the last three decades, we have seen the use of quantum computing (QC) in a variety of applications, including communication, artificial intelligence (AI), and cryptography (2). (3). Due to quantum computing's supremacy and development in addressing several issues, such as factorization by Shor's algorithm (4) and search in an unstructured database by Grover's method (5). Information processing using quantum computing, which is based on the principles and properties of quantum physics (specifically, quantum bits (Qubits), interference, superposition, and entanglement). Unlike conventional bits, which may only represent one value either 0 or 1, qubits can be in one state, zero state, or a mixture of two states at the same time known as linear superposition (6). In Hilbert space, the qubit state is a unit vector. To describe the qubit state mathematically, we utilise the ket-notation, where the qubit in state zero is |0= [1 0]. T and qubit are both equal to [0 1] in state one. T. A qubit is modelled as a simultaneous linear superposition of both base states:

$|\psi\rangle = (\alpha\beta) = \alpha|0\rangle + \beta|1\rangle$ (1)

where | 2 | + | 2 | = 1 and the coefficients and are probability amplitudes that may be complex values. Recently, several machine learning (ML) algorithms and methods have been developed, depending on quantum computing rules to present the new idea known as quantum machine learning (QML)) with the aim of improving conventional ML (8). Beyond a paradigm, QML was present. the first paradigm, quantum ML, proposed a quantum version of the support vector machine (QSVM) for the classification of huge data in 2014. Rebentrost, p. et al. Compared to its traditional equivalents, QSVM achieved logarithmic speedup. The second paradigm, quantum inspired machine learning, was proposed in 2019 by Sergioli et al. (Proposed a binary quantum classifier that was inspired by the formalisation of quantum theory and that performed better than several conventional models. In 2019, Havlicek et al. (introduced a hybrid quantum-classical machine learning (ML) paradigm that relies on the notion of the variational quantum circuit and a quantum-classical classifier. Quantum Neural Networks (QNNs) have also given rise to a number of models, including quantum inspired NNs, quantum multi-perceptron NNs, quantum dot NNs, and quantum convolutional NNs. All of these paradigms will be described in depth in section 2 along with comparisons between them. QML is utilised in many areas, including the categorization of medical data (, recommendation systems, and big data processing).

The heart and soul of quantum and QML algorithms are quantum subroutines. The performance of algorithms like Grover's algorithm based on amplitude amplification and HHL algorithms (for solving "linear system equations" in exponential speed up based on quantum phase estimation (QPE) and quantum matrix-inversion subroutines) is greatly accelerated and improved by quantum subroutines. Sampling, quantum annealing, and quantum Fourier transform are examples of additional procedures (QFT).

The primary goals of this survey are to propose a QML-based categorization method. The three basic steps (13, 14) of implementing a classical algorithm on a quantum computer or, more generally, any quantum method is encoding, quantum computation, and decoding. The encoding step, which entails mapping data from classical to quantum states, comes first. Quantum computing is dependent on type QML algorithms in the second stage. the last stage, the decoding stage. To address the query of what is the classification scheme using quantum machine learning, in addition to demonstrating the key stages of quantum classification problems?

We'll start a conversation regarding two QML algorithmic questions. We go toward quantum machine learning (QML) for what reasons? How can conventional machine learning benefit from the idea of quantum computing? The primary goals of this study are:

- Summarize and arrange the most current study findings to pave the road for quantum researchers.
- supervised machine learning
- Analyze methodologies and demonstrate the most effective and prevalent categorization methods for real-world issues.
- Provide the readership with many quantum strategies to improve classical ML and a few of the quantum techniques.
- subroutines.
- Incorporate a quantum categorization system into the quantum universe.
- Discuss some of the difficulties, potential prospects, and uses of quantum machine learning.

Literature Review:

Here, we offer extensive prior research on QML. In addition, we categorise quantum machine learning methods as QML, quantum-inspired ML, and hybrid quantum-classical ML. The categorization of techniques based on information processing devices (a sort of algorithm) as classical or quantum and the classification of data as classical or quantum are both depending on whether the data is classical or quantum (15, 16).

Montanaro, A. (17) presented an overview of quantum algorithms and their applicability in several fields. Jeswal, S. et al. (18) showed several quantum neural network approaches and their real-world applications. In contrast to traditional neural networks, QNNs are more powerful and improve computing efficiency, according to the scientists. M. Benedetti and colleagues (19) provided an overview of hybrid quantum classical models based on parameterized quantum circuits and the use of hybrid systems in supervised and generative learning. In addition, they developed a framework for model components, such as the variational circuit and encoder circuit. In (20), the authors provide a summary of the development and uses of quantum computing. In addition, the authors examined several quantum technologies for the scalability of quantum computers, such as error correction and the future. C. Ciliberto et al. (21) offered an overview of QML and its obstacles. In addition, the authors presented quantum subroutines, including quantum linear algebra and how a quantum computer processes data. In addition, this review depicts Quantum neural networks.

A. Quantum Machine Learning Algorithms:

Quantum versions of ordinary machine learning algorithms constitute the first method, quantum machine learning algorithms. as well as algorithms executable on the actual quantum gadget. Dennis, et al. (22) built SVM on quantum annealer device (23) (DW2000Q) dubbed QA-SVM. Quantum annealing was utilisedby the authors to train and optimize SVM relies on the QUBO equation to reduce energy costs. In addition, the authors employed several features of quantum annealing (reverse annealing and unique annealing schedules) to enhance the final outcomes. Rebentrost, P., et al. (24) described an SVM method that operates on a quantum computer and relies on a QSVM matrix that is not sparse. QSVM is a binary classifier for huge data. In addition, it works with a high number of features and samples with logarithmic complexity. Da Silva, A., et al. (25) presented the "quantum perceptron over a field" (QPF) quantum neural network and associated learning method (SAL). The learning algorithm (SAL) relies on quantum operator and superposition characteristic. In addition, it executes NN architecture in polynomial time. QPF overcomes the constraints of quantum perceptron models. The authors presented quantum linear regression as a variant of linear regression. It operates in logarithmic time on quantum data with N-dimensions of features.

B. Quantum-Inspired Machine Learning:

The second strategy is quantum-inspired machine learning, which utilises the concepts of Quantum Computing (QC) to enhance traditional machine learning techniques (ML).

Prayag et al. (27) proposed a novel quantum-inspired binary classifier (QIBC) based on decision theory, classical ML, and the theory of quantum detection that employs one of the rules of quantum mechanics, superposition, in order to improve the degree of freedom in decision making. The suggested classifier achieves equivalent accuracy, recall, and F-measure to KNN, SVM, and other traditional approaches. Sergioli et al. (28) developed the Helstrom Quantum Centroid (HQC), a new quantum-inspired classifier for binary supervised learning based on density matrices and the formalism of quantum theory. The authors compared the performance of their model to other classical models using fourteen datasets. Ding et al. (29) introduced a new approach inspired by quantum support vector machines (SVM) to handle exponentially fast classification tasks. The fundamental concept behind the linear transformation-based algorithm

Sergioli et al. (30) developed the Quantum Nearest Mean Classifier (QNMC) based on the conventional minimum distance classifier. First, a density pattern (Encoding) is applied to each classical data point in order to convert it into a quantum entity. Quantum centroid is then used to compute the distance between density patterns in order to categorise unknown quantum items into the correct category. decoding to convert the final classification result into the classified data In some medical data sets, the algorithm obtained more precision than its conventional equivalent (NMC), which was limited to cancer data. Y. Dang et al. (31) introduced a novel model for image classification based on Quantum KNN and parallel computing. Their model improves classification performance and efficiency. Chen, H. et al. (32) used strong parallel computing to present an inspired Quantum K Nearest-Neighbor (QKNN) approach based on one of the well-known aspects of quantum computing, superposition, to acquire parallel computing and "quantum minimum search algorithm" to speed up the search.

C. Hybrid Quantum-Classical Machine Learning:

The last option, hybrid quantum-classical machine learning, utilises algorithms that blend quantum and classical (conventional) algorithms to improve performance and reduce the cost of learning. Using the quantum circuit, Soumik et al. (38) presented a novel variational quantum classifier using a single quantum system (Qu N it) to encode N-dimensional input using a training approach known as "single-shot training." The primary benefit of single-shot training is that it requires fewer training parameters and achieves greater accuracy. In (39), the authors developed a novel quantum algorithm based on various subroutines such as a quantum oracle, counting, amplitude amplification, and quantum amplitude estimation for feature selection called (HQFSA) with the goal of improving the performance of machine learning (ML) approaches. In certain instances, the suggested approach achieved quadratic time complexity and enhanced performance. The primary drawback of HQFSA is that it runs only on quantum simulators.

Havlicek et al. (40) proposed two distinct quantum support vector machine models. The first is a quantum variational SVM based on a quantum variation circuit. To classify, the variational quantum SVM classifier needs two algorithms. In the one-to-quantum variational training phase, the authors used four stages to calculate the hyperplane between training data and fresh data to assign the right label. The second model is a quantum kernel estimation-based SVM quantum kernel-based method. For classification challenges, Maria Schuld et al. (41) presented two- hybrid quantum approaches. Schuld demonstrated that quantum computing improves standard ML techniques such as kernel approaches. Quantum computing conducts more efficient complicated calculations in Hilbert space. The authors focused on the use of feature maps and kernel approaches in the realm of quantum computing.

Mitarai et al. (42) introduced a hybrid quantum-classical approach for performing diverse tasks, such as classification, regression, and clustering, using tiny quantum devices. In her Ph.D. dissertation, Jessica Pointing presented (43) a revolutionary quantum-classical method for handling missing values in data. This technique is titled "a Quantum Algorithm for Handling Missing Data." The primary benefit of the technique is its ability to construct the probability distribution for missing data values.

Challenges and Future Directions:

In this part, we discuss and present obstacles and future prospects in QML, such as the development of a small-scale quantum computer, limited quantum bits, encoding approaches, and novel QML techniques.

1. Small-Scale Quantum Computer:

In the near future, it will be crucial to construct quantum computers with a huge number of qubits in order to create QML algorithms, test them, and process vast amounts of data. Figure 3 depicts the number of qubits attained by 75 various technological firms, including Rigetti, IBM, Q-Wave, Xanadu, Google, and Microsoft. Until recently, the quantum computer has been built on a modest scale, limiting our ability to use a restricted quantity of data (76). So, researchers design algorithms suitable with the number of qubits on existing small-scale and "noisy intermediate-scale quantum" (NISQ) technology (77). Quantum devices with a restricted number of qubits cannot do large-scale data processing due to the loss of a large amount of vital information caused by the use of fewer features (78, 79).

Figure 1. A chart shows the progress of quantum computer with numbers Quantum Bits: Source (75),

Quantities of qubits attained by various firms. IBM, the MIT Media Lab, and UC Berkeley constructed the first quantum computer with two qubits in 1997.IBM reached 50 qubits in 2017; it took 20 years to expand 48 qubits.in 2019, Rigetti achieved 128 qubits; it took just two years to increase 78 qubits. If the number of qubits increases with a high mistake rate, a strong quantum processor cannot be created (80). The power of a quantum processor with a low error rate will expand exponentially as the number of quantum bits increases.

2. Encoding Methods:

Encoding data to quantum states is one of QML's obstacles; this procedure requires a great deal of time and energy to transfer classical data to quantum data (74) (i.e., image and big data). Therefore, designing new ways for data encoding is a promising area of future study. In 2020, LaRose et al (81).Presented a binary quantum classifier that is resilient to noisy quantum states based on the optimal encoding strategy for loading input into the quantum system. In addition, the authors explored several encoding strategies using quantum binary classifier. Additionally, the authors used many encoding data strategies to the same data and shown that encoding data techniques may increase model precision. In 2020, the authors (74) proposed a novel approach for compressing vast amounts of picture data into a small number of qubits using quantum annealing computer technology. In addition, the authors trained a limited Boltzmann machine to categorise picture data.

3. A New QML Techniques:

In the near future, we anticipate that academics will create new theoretical and practical machine learning (ML) algorithms that are compatible with existing quantum technology. use quantum information theory or quantum subroutines to solve machine learning difficulties and enhance performance. Create variants of existing algorithms in many disciplines. Quantum neural networks (QNNs), quantum deep learning, quantum-inspired machine learning, and quantum-enhanced machine learning (82), as well as the development of novel classification algorithms based on quantum "variational circuit" and the reduction of the depth of quantum circuit. In addition, the application of classical approaches to actual quantum devices or simulators. Combining quantum annealing and adiabatic computing with conventional machine learning to create new QML paradigms and address complex optimization challenges linked to ML difficulties.

Applications:

Quantum Machine Learning (QML) techniques are superior to traditional machine learning in many real-world applications, including big data classification, forecasting series, spam detection, image compression, and medical domain (30,43,37) (e.g., cervical cancer detection (83), electronic calculations (84), decision games (85), natural language processing (NLP), recommendation systems (86, 87), speech recognition (88), and image classification (89)).

Conclusion:

This overview study examined the most current publications on quantum machine learning strategies for a variety of issues that are superior to standard techniques for conventional computing in their ability to efficiently and precisely manage a variety of difficulties. In addition, we compared quantum-inspired algorithms with hybrid quantum-classical algorithms. according to Section 2. (Review of relevant literature) QML algorithms beat traditional ML in terms of performance and speed. In addition, we provided the quantum scheme for classification problems and supervised learning in general, and then we outlined two strategies for data mapping. Quantum enhancements to machine learning, such as quantum subroutines, were considered. We showed applications of QML in many sectors and our vision for the future in order to open up new paths for the use of quantum machine learning in the field of research. In our future study, we will compare the performance of quantum-inspired and hybrid quantum-classical algorithms on real-world data.

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