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A THEORY KNOWN AS THE GENERALIZED INTERVAL VALUED INTUITIONISTIC FUZZY SETS

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Abstract:

In this research, we introduce a new generalised interval-valued intuitionistic fuzzy sets (GIVIFS), which is the generalisation of conventional intuitionistic fuzzy sets (IFS) and interval-valued intuitionistic fuzzy sets. In other words, GIVIFS is the generalisation of conventional IFS (IVIFS). This study presents generalised interval-valued intuitionistic fuzzy sets with parameters by doing an analysis of the degree of reluctance (GIVIFSP). After that, it is shown that GIVIFS, in addition to IFS and IVIFS, is a closed algebraic system. Atanasov first introduced IFS in the year 1986. Atanassov first presented the IVIFS algorithm in 1989, basing it on a comparison of the interval-valued fuzzy sets (IVVS) and the IFS algorithm. As a result, a great number of academics extensively employed IFS and IVIFS to decision analysis and pattern identification.

Keywords: intuitionistic fuzzy sets; interval-valued intuitionistic fuzzy sets; generalized interval-valued intuitionistic fuzzy sets; interval-valued intuitionistic fuzzy sets with parameters

Introduction:

Atanasov first introduced IFS in the year 1986. Atanassov first presented the IVIFS algorithm in 1989, basing it on a comparison of the interval-valued fuzzy sets (IVVS) and the IFS algorithm. As a result, a great number of academics extensively employed IFS and IVIFS to decision analysis and pattern identification. In the field of IVIFS research, Yager, Yuan Xuehai, and Li Hongxing discussed the cut set characteristics of IVIFS in [5, 6, 7], and Xu Zeshui and Zhang Qianshengapplied it to pattern recognition based on [4] in [8, 12, 13]. Both of these studies can be found in [5, 6, 7], [8, 12, 13], and [8, 12, 13]. It was also used to decision-making analysis by Xu Zeshui and Li Dengfeng in [9, 10, 11], while Lei Yingjie and Zhang Qiansheng conducted research on interval-valued intuitionistic fuzzy reasoning in [14, 15].

The IFS theory and the IVIFS theory are both created, which generalises Zadeh's fuzzy sets (FS) by incorporating the degree of membership MA(x), the degree of non-membership NA(x), and the degree of hesitation HA(x). These degrees are denoted by the symbols MA(x), NA(x), and HA(x) ([1, 2, 3]). MA(x), NA(x), and HA(x) are all considered to be intervals in the context of the IVIFS definition. More specifically, MA(x) refers to the range of support for a party, NA(x) refers to the range of opposition for a party, and HA(x) refers to the range of missing support for a party. In addition to this, the inferior of MA(x) (INF(MA(x))) is the firm support party of event A, the inferior of NA(x) (INF(NA(x))) is the firm opposition party of event A, the inferior of HA(x) (INF(HA(x))) is the firm absent party of event A, the superior of HA(x) (SUP(HA(x))) is the maximum absent party of event A, and SUP Atanassov has split the convertible absent part into two parts: SUP (MA(x))-INF (MA(x)), which is the absent party that can be converted into the support party, and SUP (NA(x))-INF (NA(x)), which is the absent party that can be converted into the opposition party, with SUP (MA(x))-INF (MA(x)) plus SUP (NA(x))-INF (NA(x)) equal Since Atanassov's IVIFS is based on point estimate, this indicates that these intervals may be viewed as the estimation result of an experiment. In other words, these intervals can be thought of as a measure of precision. However, there is a possibility that the proportions of the missing party that are converted to the supporting party and the opposing party will not remain consistent. For instance, the difference between SUP (MA(x)) and INF (MA(x)) is a constant for one experiment, but it may be a completely different constant for any other case. Therefore, in accordance with interval estimate, we provide a fresh GIVIFS model in order to satisfy actual need.

To begin, we will introduce the idea of GIVIFS, which has been shown to be the generalisation of IFS and IVIFS. After that, we provide the building technique of the generalised interval valued intuitionistic fuzzy sets with parameters (GIVIFSP), and then we describe the complement operation, the intersection operation, and the union operation on GIVIFS. Finally, we show that GIVIFS is a closed algebraic system for all of these operations, including fuzzy sets, IFS, and IVIFS, by proving that GIVIFS can be represented as a fuzzy set. Therefore, this article generalisesthe IVIFS theory, and it gives some relevant results for the area of application research of IVIFS. Additionally, this study is beneficial to the generalisation of interval-valued intuitionistic fuzzy reasoning.

Construction of GIVIFS:

Definition 1. An IFS A in universe X is given by (Atanassov [1, 2, 3]):

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \square X\} \quad (1)$$

where $\mu_A : X \rightarrow [0, 1], \nu_A : X \rightarrow [0, 1]$ with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \square X$. The numbers $\mu_A(x), \nu_A(x) \square [0, 1]$ denote the degree of membership and the degree of non-membership of x to A, respectively. For each IFS in X, we call $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ the degree of hesitancy ([9]) of x to A, $0 \leq \pi_A(x) \leq 1$ for each $x \square$ X.

Definition 2. An IVIFS A in universe X is given by (Atanassov [2, 3]):

$$A = \{< x, M_A(x), N_A(x) > | x \square X\} \quad (2)$$

where $M_A(x)=[t_A^-(x), t_A^+(x)]$, $N_A(x)=[f_A^-(x), f_A^+(x)]$, and $H_A(x) = [\pi_A^-(x), \pi_A^+(x)]$ with the condition $M_A(x) \subseteq [0,1], N_A(x) \subseteq [0,1], P_A(x) \subseteq [0,1]$. The interval $M_A(x)$, $N_A(x)$, and $H_A(x)$ denote the degree of membership, the degree of non-membership, and the degree of hesitancy of x to A, respectively. For each IVIFS in X, we call $\pi_A^-(x) = 1 - t_A^+(x) - f_A^+(x)$, $\pi_A^+(x) = 1 - t_A^-(x) - f_A^-(x)$ lower bound and upper bound of hesitancy of x *to* A respectively, where $0 \leq \pi_A^-(x) \leq \pi_A^+(x) \leq 1$, for each $x \square X$.

Theorem1. Suppose that A is an IVIFS as mentioned above, then

$$t_A^+(x) - t_A^-(x) + f_A^+(x) - f_A^-(x) = \pi_A^+(x) - \pi_A^-(x). \quad (3)$$

Based on definition2, we have (3).

From definition 2, let all samples be divided into three parts, $t_A^-(x)$ being firm support party of event A, $f_A^-(x)$ representing firm opposition party of event A, and $\pi_A^+(x)$ showing all absent party. In

absent party, $\pi_A^-(x)$ is firm absent party, and $\pi_A^+(x) - \pi_A^-(x)$ is convertible absent party, in which each sample may become one of the support party and the opposition party. Suppose that there is $t_A^+(x) - t_A^-(x)$ of the samples supporting event A and $f_A^+(x) - f_A^-(x)$ of the samples opposing event A, and then

$$0 \leq t_A^+(x) - t_A^-(x) \leq \pi_A^+(x) - \pi_A^-(x), 0 \leq f_A^+(x) - f_A^-(x) \leq \pi_A^+(x) - \pi_A^-(x).$$

Let $\alpha_A(x) = t_A^+(x) - t_A^-(x), \beta_A(x) = f_A^+(x) - f_A^-(x)$, and we will get the GIVIFS definition as follows:

Definition 3. Let X be a universe of discourse. A GIVIFS A in X is an object having the form:

$$A=\{<x, M_A(x), N_A(x)>|x \in X\}$$

where the intervals $M_A(x)$, $N_A(x)$, $H_A(x)$ are the same as definition 2. Let $t_A^+(x) = t_A^-(x) + \alpha_A(x)$ and $f_A^+(x) = f_A^-(x) + \beta_A(x)$, then $0 \leq \max\{\alpha_A(x), \beta_A(x)\} \leq \pi_A^+(x) - \pi_A^-(x)$. Obviously, if $\pi_A^+(x) = \pi_A^-(x) = 0$, then $\alpha_A(x) = \beta_A(x) = 0$ and GIVIFS is fuzzy sets; and if $\alpha_A(x) = \beta_A(x) = 0$, then GIVIFS is IFS; and if $\alpha_A(x) + \beta_A(x) = t_A^+(x) - t_A^-(x) + f_A^+(x) - f_A^-(x) = \pi_A^+(x) - \pi_A^-(x)$, then GIVIFS is IVIFS.

Suppose that the proportion of absent party converted to the support party is $\lambda_{A1}(x)$ and that converted to the opposition party is $1 - \lambda_{A1}(x)$. The model will become interval valued intuitionistic fuzzy sets with single parameter, where

$$\alpha_A(x) = \lambda_{A1}(x)(\pi_A^+(x) - \pi_A^-(x)), \beta_A(x) = (1 - \lambda_{A1}(x))(\pi_A^+(x) - \pi_A^-(x)).$$

Definition 4. Let X be a universe of discourse. A GIVIFSP A in X is an object having the form:

$$A=\{<x, M_A(x), N_A(x)>|x\in X\}$$

where $M_A(x)$, $N_A(x)$, and $H_A(x)$ are the same as definition3. Let

$t_A^+(x)=t_A^-(x)+\lambda_{A0}(x)\lambda_{A1}(x)\pi_A^+(x), f_A^+(x)=f_A^-(x)+\lambda_{A0}(x)\lambda_{A2}(x)\pi_A^+(x), \pi_A^-(x)=(1-\lambda_{A0}(x))\pi_A^+(x),$

where $0\le\lambda_{Ai}(x)\le 1, i=0,1,2$. $M_A(x)$, $N_A(x)$, And $H_A(x)$ represent the degree range of membership, the degree range of non-membership, and the degree range of hesitancy of x to A, respectively.

It is clear that we will get the following conclusions: If $\pi_A^+(x)=\pi_A^-(x)=0$, then $\lambda_{A0}(x)=0$, and then GIVIFS is fuzzy sets; and if $\lambda_{A0}(x)=0$, then GIVIFS is IFS; and if $\lambda_{A0}(x)>0$ and $\lambda_{A1}(x)+\lambda_{A2}(x)=1$, then GIVIFS is IVIFS. Furthermore, if $\lambda_{Ai}(x)=\lambda_i, i=0,1,2$, and λ_i is constant, then GIVIFSP is an interval valued intuitionistic fuzzy sets with fixed parameters, otherwise it is a variable model.

Algebraic Properties of GIVIFS:

Following is a presentation of the fundamental GIVIFS operations consisting of containment relation, equal relation, intersection, union, and complement.:

Definition 5. Let X be a universe of discourse. A and B are two GIVIFSPs in X. $A=\{<x, M_A(x), N_A(x)>|x\in X\}$, $B=\{<x, M_B(x), N_B(x)>|x\in X\}$, where $M_A(x)$, $N_A(x)$, and $H_A(x)$ are the same as definition 3.

(1). $A\subseteq B \Leftrightarrow M_A(x)\subseteq M_B(x), N_B(x)\subseteq N_A(x) \Leftrightarrow t_A^-(x)\le t_B^-(x), t_A^+(x)\le t_B^+(x), f_A^-(x)\ge f_B^-(x), f_A^+(x)\ge f_B^+(x);$

(2). $A=B \Leftrightarrow A\subseteq B, B\subseteq A \Leftrightarrow M_A(x)=M_B(x), N_B(x)=N_A(x) \Leftrightarrow t_A^-(x)=t_B^-(x), t_A^+(x)=t_B^+(x), f_A^-(x)=f_B^-(x), f_A^+(x)=f_B^+(x);$

(3). $A\cap B=\{<x,M_A(x),N_A(x)>|x\in X\}\cap\{<x,M_B(x),N_B(x)>|x\in X\}$

$=\{<x,[t_A^-(x),t_A^+(x)],[f_A^-(x),f_A^+(x)]>|x\in X\}\cap\{<x,[t_B^-(x),t_B^+(x)],[f_B^-(x),f_B^+(x)]>|x\in X\}$

$=\{<x,[t_A^-(x)\wedge t_B^-(x),t_A^+(x)\wedge t_B^+(x)],[f_A^-(x)\vee f_B^-(x),f_A^+(x)\vee f_B^+(x)]>|x\in X\};$

(4). $A\cup B=\{<x,M_A(x),N_A(x)>|x\in X\}\cup\{<x,M_B(x),N_B(x)>|x\in X\}$

$=\{<x,[t_A^-(x),t_A^+(x)],[f_A^-(x),f_A^+(x)]>|x\in X\}\cup\{<x,[t_B^-(x),t_B^+(x)],[f_B^-(x),f_B^+(x)]>|x\in X\}$

$=\{<x,[t_A^-(x)\vee t_B^-(x),t_A^+(x)\vee t_B^+(x)],[f_A^-(x)\wedge f_B^-(x),f_A^+(x)\wedge f_B^+(x)]>|x\in X\};$

(5). $A^C=\{<x,M_A(x),N_A(x)>|x\in X\}^C=\{<x,N_A(x),M_A(x)>|x\in X\}=\{<x,[f_A^-(x),f_A^+(x)],[t_A^-(x),t_A^+(x)]>|x\in X\}.$

Theorem2. GIVIFS is closed for complement operation, containment operation, and union operation.

Proof: Support that A and B are GIVIFSs. We will prove that A^C, $A\cap B$, and $A\cup B$ are also GIVIFSs.

$A^C=\{<x,M_A(x),N_A(x)>|x\in X\}^C=\{<x,N_A(x),M_A(x)>|x\in X\}=\{<x,[f_A^-(x),f_A^+(x)],[t_A^-(x),t_A^+(x)]>|x\in X\}$

$=\{<x,[f_A^-(x),f_A^-(x)+\beta_A(x)],[t_A^-(x),t_A^+(x)+\alpha_A(x)]>|x\in X\}, 0\le\max\{\alpha_A(x),\beta_A(x)\}\le\pi_A^+(x)-\pi_A^-(x).$

Thus, A^C is also GIVIFS.

According to definition 3, $\pi_{A\cap B}^+(x),\pi_{A\cap B}^-(x),\pi_{A\cup B}^+(x),\pi_{A\cup B}^-(x)$ can be defined as $\pi_A^+(x),\pi_A^-(x)$. Obviously,

$0\le t_A^+(x)-t_A^-(x)\le\pi_A^+(x)-\pi_A^-(x), 0\le f_A^+(x)-f_A^-(x)\le\pi_A^+(x)-\pi_A^-(x), 0\le t_B^+(x)-t_B^-(x)\le\pi_B^+(x)-\pi_B^-(x),$

$0\le f_B^+(x)-f_B^-(x)\le\pi_B^+(x)-\pi_B^-(x), \pi_{A\cap B}^+(x)=1-t_{A\cap B}^-(x)-f_{A\cap B}^-(x), \pi_{A\cap B}^-(x)=1-t_{A\cap B}^+(x)-f_{A\cap B}^+(x),$

Conclusion:

We put out a definition for GIVIFS and shown that it is a generalisation of IFS and IVIFS. As a result, we will describe GIVIFSP and then build a specific kind of GIVIFSP model. We conclude by demonstrating that GIVIFS is a closed algebraic system for the complement operation, the intersection operation, and the union operation when applied to fuzzy sets, IFS, and IVIFS.

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