International Journal of Advance and Applied Research
www.ijaar.co.in
ISSN – 2347-7075 Impact Factor – 7.328
Peer Reviewed Bi-Monthly
Vol.10 No.3 January – February 2023

CONCERNING THE EVALUATION AND CALCULATION OF TOPOLOGICAL FUZZY MEASURE IN DISTRIBUTED MONOID SPACES

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DOI - 10.5281/zenodo.7895780

Abstract:

The computational uses of fuzzy sets are widespread in systems that include intrinsic uncertainties and multivalued logic-based approximations. The current fuzzy analytic measures are constructed from fuzzy topological spaces and are dependent on regularity variations. An investigation into the generic fuzzy measures in n-dimensional topological spaces with monoid embeddings is presented as a proposition in this work. The topological distribution of the embedded monoids may be seen across the measure space. There will be a presentation on the analytical characteristics of compactness and homeomorphic, as well as isomorphic mappings between spaces. The computational assessments are done with n = 1, taking into consideration a variety of translation functions with varying symmetry profiles. In a monoid topological subspace, the findings demonstrate the dynamics of finite fuzzy measure.

Keywords: fuzzy sets, fuzzy measures, monoid topological Subspace, compactness, computational application

Introduction:

The fuzzy set theory may be used in a wide variety of different kinds of complicated systems in a number of different ways. The use of fuzzy sets allows for decision making in information systems even in the face of uncertainties or incompleteness [1–3]. When modelling the inherent uncertainties and imprecision of information, it is possible to use ordered fuzzy numbers, with the fuzzy ordering being constructed by utilising trapezoidal structures [4]. This allows for a more accurate representation of the situation. In most cases, the process of making decisions using fuzzy logic requires calculations to be performed on multi-granular rough fuzzy sets [2]. The effectiveness of information aggregation and filtering is directly related to the granularity of the data being processed. The convergence theory may be used to do an analysis of fuzzy inference procedures, which can then be employed in a variety of topological contexts. In general, the development of fuzzy sets and fuzzy topology are produced based on relational algebra, on a crisp set with an upper limit and lower limit [1]. This is how fuzzy sets and fuzzy topology are formed.

The hybridization of fuzzy sets, lattices, and topology has led to the construction of many diverse structures of fuzzy topological spaces, each of which may be interpreted in a unique way. The fuzzy set theory is used to topology in one strategy in order to generate fuzzy topological spaces [5]. This is only one of many possible approaches. The structures of L-topology, which are based on lattice models, have been established as a result of this formulation. Alternatively, the topological spaces may be built by using the L-subset of the power set of any arbitrary set X [6]. This method is used to do this. Because of this, the topology that is based on the L-subset is referred to as the L-fuzzy topology, and it has a unique structure in comparison to the L-topology [7]. There is a connection that can be made between the convexity of the underlying spaces and the fuzzy sets that are being considered. A continuous and complete lattice is used as the foundation for the formulation of the L-convex spaces [8]. The embedding of a wide variety of algebraic structure categories is made possible by the L-convex space. In order to properly formulate comparable metrics, it is necessary to conduct an examination of the compactness of fuzzy sets as well as topological spaces [9]. It is possible to do an analysis on the level of compactness and countability that exists in L-fuzzy pretopological spaces [10]. In order to formulate pretopological spaces, a non-distributive lattice and implication operator algebra have been combined. This combination serves as the foundation. Recently, the graph theoretic approach of the topology has been employed for the purpose of understanding the physical lattice-like structures of the honeycomb networks of physical systems [11]. In topologically described graph-like structures, the topological indices and corresponding connectivity polynomials are the parameters that remain constant no matter what changes are made.

The fuzzy topological space specifications are improved by including two functorsin order to build a category that maintains the fuzzy compactness [12]. This category is then used to construct the fuzzy topological space. Based on the sets of probability measures present in a supported space, the idea of a fuzzy ultrametric is presented [13]. A fuzzy metric space with a compact support is used in the construction of the measure. [14] presents a formulation of the measures of noncompact fuzzy subsets in conventional as well as fuzzy metric spaces. It is possible to preserve the consistency of the various measures by keeping the regularity of the null-additivity of fuzzy measures on a metric space [15]. The fuzzy measure may be created on any arbitrary topological space by using the generic Borel sets as the foundation [16]. The measure maintains the monotone class of the Borel sets, but it is not guaranteed to be finite in every circumstance. From an algebraic standpoint, the measure that is formed on an Abelian group and given the name the Haar measure [17] is referred to as the Haar measure. It is generally accepted that the natural environment has locally compact Haar measurable groups. The commutative convolution measure algebra is isomorphic to L 1 (G), and it has a representation that is finite-dimensional involution [18]. If a space cannot be separated into two halves, then there must be an infinite number of pairwise disjoint subsets. These subsets must be open and measurable on any locally compact Abelian group.

Preliminaries:

Let X be any arbitrary set equipped with a binary relation (or operation) $*: X^2 \to X$. The structure $(X, *)$ is called a group if the following axioms are maintained [20]:

$$\begin{aligned} &\forall x, y, z \in X, x * (y * z) = (x * y) * z, \\ &\exists e \in X : \forall x \in X, e * x = x * e = x, \\ &\forall x \in X, \exists x^{-1} \in X : x * x^{-1} = x^{-1} * x = e \end{aligned} \tag{1}$$

The group $(X, *)$ is called Abelian if $\forall x, y \in X, x * y = y * x$. If X represents an underlying space, then a corresponding topology $\tau \subseteq P(X)$ can be constructed on it. The topology τ should maintain the following axioms (I represents the index set) [21]:

$$\begin{aligned} &\forall A_i \in \tau, i \in I, \bigcup_{i \in I} A_i \in \tau, \\ &\forall A_i, A_k \in \tau, A_i \cap A_k \in \tau, \\ &\{\phi, X\} \subset \tau \end{aligned} \tag{2}$$

A space is called a sigma measurable if it can be equipped with a function, $\mu : (A_i \subset X) \to [0, +\infty]$. A measure is finite if $\forall A_i \subset X, \mu(A_i) < +\infty$. A fuzzy set is a set equipped with a nonbinary discrete measure, given as (X, μ_F), such that $\mu_F : X \to [0, 1]$. A fuzzy set can have a support built into it.

Definitions:

Groupoid Space:

Let X be any arbitrary point set and A ⊂ X is equipped with an operation in n-dimensional space as ox : A 2n → A n , such that ∀a, b ∈ A n , ∃c ∈ A n , where c = aoxb. The (X n , ox) is a groupoid space.

Distributed Monoid Space:

Let (X^n, o_x) be a groupoid space and $M = \{B_i \subset A^n : i \in I \subset Z^+\}$, where $B_i \cap B_k = \phi$ if $i \neq k$ and $\bigcup_{i\in I} B_i = A^n$. The M is a distributed monoid (DM) if $\forall B_i \in M$ the following properties are satisfied [20]:

$$\begin{aligned} &\forall a, b, c \in B_i, (ao_x b)o_x c = ao_x(bo_x c), \\ &\forall a \in B_i, \exists e_i \in B_i : ao_x e_i = e_i o_x a = a \end{aligned} \tag{3}$$

Analytical Properties:

Theorem 1. *In (X^n, M, τ), the topological fuzzy measure in the topological DM subspace is countably additive as $\mu_\tau(\bigcup_{i\in I} B_i) = \sum_{i\in I} \mu_\tau(B_i)$.*

Proof. Let (X^n, M, τ) be a topological DM space. Let $\forall B_i \in M, \exists \beta_i \in (1, +\infty)$, such that $\mu_\tau(B_i) = \beta_i^{-1} \sum_{\forall a \in B_i} |g(a)^{-1}|$, where $0 < \mu_\tau(B_i) < 1$. As $[i \neq k] \Rightarrow [B_i \cap B_k = \phi]$, thus

$\mu_\tau(M) = \sum_{i\in I} [\beta_i^{-1} \sum_{\forall a \in B_i} |g(a)^{-1}|]$. However, in the distributed monoid subspace, $\sum_{i\in I} \mu_\tau(B_i) = \sum_{i\in I} [\beta_i^{-1} \sum_{\forall a \in B_i} |g(a)^{-1}|]$. Hence, $\mu_\tau(\bigcup_{i\in I} B_i) = \sum_{i\in I} \mu_\tau(B_i)$ in the topological DM space (X^n, M, τ). □

Theorem 2. *In the (X^n, M, τ) space $\exists D, E \in X^n$, such that $\mu_\tau(D \cap E) = 0$ and $\mu_\tau(D \cup E) \in (0, 1)$.*

Proof. Let (X^n, M, τ) be a topological DM space, such that $\exists D, E \subset X^n$, where $\{D, E\} \subset \tau$. Let $\{D, E\} \not\subset \tau \backslash M$ and, by following topological properties, $D \cup E \in \tau$. However, this indicates that $\mu_\tau(D \cap E = \phi) = 0$ in (X^n, M, τ). On the other hand, $X^n \backslash \{D \cup E\} \neq \phi$ and $\mu_\tau(D) + \mu_\tau(E) > 0$. As $X^n \neq \phi$ and $0 < \mu_\tau(X^n) \leq 1$, thus $\mu_\tau(D \cup E) < 1$, where $g(.) \in R \backslash \{-\infty, 0, +\infty\}$. Hence, $\mu_\tau(D \cup E) \in (0, 1)$ in the topological DM space (X^n, M, τ). □

Lemma. *In the (X^n, M, τ) space, the monotonicity of the topological fuzzy measure is preserved as $\mu_\tau(\tau) > \mu_\tau(M)$.*

Proof. In the (X^n, M, τ) space, $\tau \neq \phi$, and it results in the condition that $\mu_\tau(\tau \backslash \{\phi\}) \in (0, 1]$. However, $\tau \backslash M \neq \phi$ and $\mu_\tau(\tau \backslash M) \in (0, 1)$ in the underlying topological DM space. Hence, $\mu_\tau(\tau) > \mu_\tau(M)$, preserving the monotonicity of the topological fuzzy measure in the (X^n, M, τ) space. □

Computational Evaluations:

The profile of the inverse translation is shown in Figure 1, and it demonstrates that the translation looks to have symmetric distribution and has a greater concentration zone around the origin. Figure 2's fixed coefficient translation profiles examine two different values for the coefficients, and the resultant profiles show that the changes in translations have essentially symmetric distributions over the domain. These profiles are displayed in the figure. The null (zero) translation point is not covered by either the inverse or the fixed coefficient translations (i.e., translations are always in the non-zero positive range).

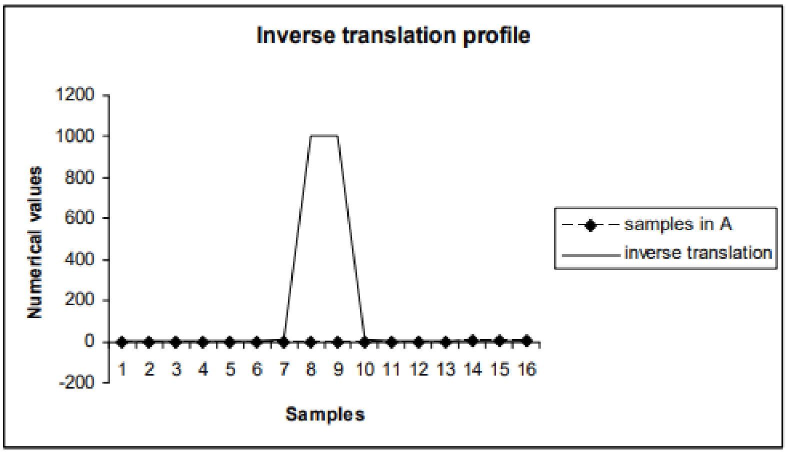

Figure 1. Profile of modulus of inverse translation.

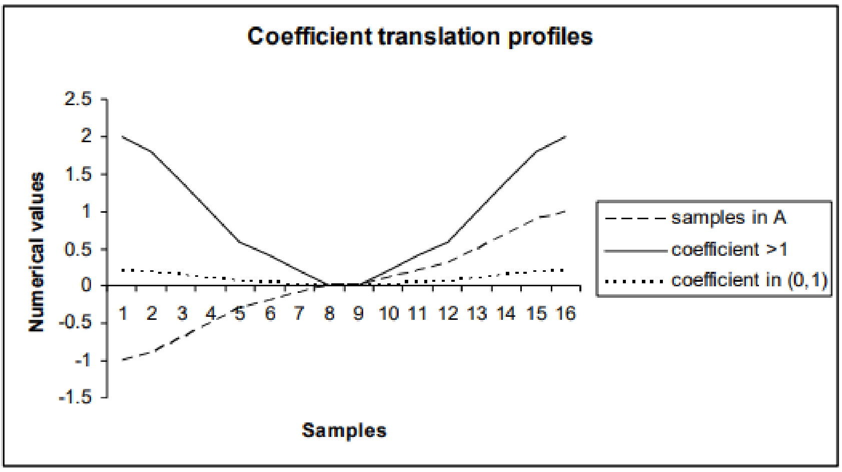

Figure 2. Profiles of modulus of multiplicative translation.

Conclusion:

The analytical knowledge of topological fuzzy measures makes it possible to create particular measure spaces, which in turn makes its applications easier to implement. When the underlying spaces include monoid algebraic structures, the analysis and calculation of topological fuzzy measures are affected in a variety of ways. In this article, the idea of topologically distributed monoid spaces is introduced, and a series of unique theorems is established in order to get analytical insights into fuzzy measures in such spaces. The results of these investigations are given and discussed. Utilizing a particular control parameter allows for the fuzzy measure of the distributed monoid spaces to be calculated with a variety of distribution profiles that span the whole of the measure range. The measures have a characteristic of being finite, and it is possible to define comparable topological fuzzy measures in two locally homeomorphic monoid spaces provided that the underlying space is of the Hausdorff type. The topological fuzzy measure that has been suggested maintains its consistency even when compressed. The limiting unity measure is maintained in compact spaces by the covering topological fuzzy measure of the distributed monoids. In addition, under some circumstances, the minimum form of the fuzzy Haar measurability may be maintained in topologically distributed monoid spaces. Only identity components are taken into consideration while calculating the minimum Haar measurability of the proposed topological fuzzy measure. The suggested measure does not take into account the presence of a group structure in the underlying space. This is the most restrictive possible interpretation of the statement. It is possible to calculate the topological fuzzy measures in the distributed monoid spaces under a variety of translations, which results in a wide range of measure distributions and covers.

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