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Non-Classical Property of Radiation Field

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Abstract:

The thesis explores the quantum properties of electromagnetic fields. It is focusing on the transition from classical optics to quantum optics. From the quantization of the electromagnetic field based on Maxwell's equations, it studies photon statistics in thermal and coherent fields. The study draws a comparison between the classical Bose-Einstein statistics of thermal fields and the Poissonian statistics of quantum coherent states (like laser light).

The main focus of the study is on non-classical light, including squeezed states and photon anti-bunching. Squeezed states, where noise is reduced in one quadrature. This phenomenon is used for detecting high-precision measurements, like gravitational waves. Photon anti-bunching is a property of single-photon sources, reveals the quantum particle-like nature of light. The study explorers the significance of these non-classical states in quantum information science, which are highly used in teleportation, cryptography and computing.

Keywords: Squeezed state, Photon anti-bunching, Coherent state

Introduction

In the last decade we have seen that people have surge of interest in investigating quantum statistical properties of opto- micro- or nano-mechanical system. People have put lots of effort to prepare entangled or non-classical states of such mechanical system.

In 1980 Hanbury Brown and Twiss first time measured the intensity correlation in a light beam in order to build an improve version of Mickleson steller interferometer. From then a new subject comes which is nowadays known as Quantum optics.

There are many applications of non classical light. For example, squeezed state are use to detect gravitational cryptography, teleportation of coherent states etc. photon anti-bunching is used to build single photon source and Entangled state which is one of the main source of Quantum Information theory.

This article examines the features of radiation fields and reviews theoretical research that has been done on the issue of radiation field creation.

Electromagnetic field quantization

In order to discuss the quantization of the electromagnetic field, we start with the well known Maxwell's equation for electrodynamics. The corresponding equations in MKS system are given by-

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{....... (1)}$$

$$\vec{\nabla} \times \vec{H} = \vec{\iota} + \frac{\partial \vec{D}}{\partial \vec{t}} \quad \text{..... (2)}$$

$$\vec{\nabla} \cdot \vec{D} = \rho \quad \text{..... (3)}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \text{..... (4)}$$

Where i = Current density, ρ = Charge density,
$\vec{E}(r,t)$ =Electric field, $\vec{B}(r,t)$ = Magnetic field, $\vec{D}$ = Displacement vector $\vec{H}$ = Magnetic field vector.
By using a small algebra we end up with the following equation of continuity-

$$\vec{\nabla} \cdot \vec{\iota} = -\frac{\partial \rho}{\partial t} \quad \text{...... (5)}$$

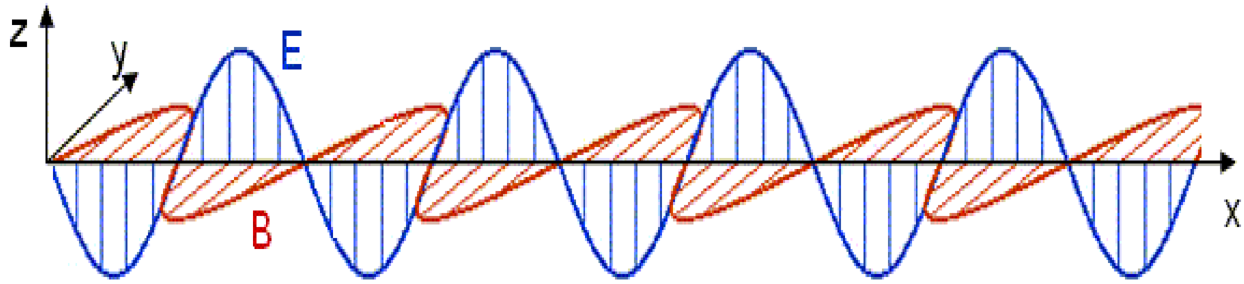


Fig-1

For charge free and current free situation, the solution of the Maxwell's equation give rise to Electric and Magnetic field. Nevertheless, this fields are transverse in nature. The Fig-1 exhibits the transverse nature clearly.

Now we consider a cavity whose surface is perfectly conducting in nature. It corresponds the region is homogeneous, charge free and isotropic. Therefore we have-

$$\vec{\imath} = 0, \vec{\rho} = 0.$$

Now we can write –

$$\vec{\nabla} \cdot \vec{D} = 0$$

$$\vec{D} = \in \vec{E}$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

Where $\in = Dielectric\ constant.$

Consider the electric field $\vec{E}(r,t) and\ magnetic\ field\ \vec{H}(r,t)$ inside a volume 'v' and bounded by a surface's' of perfect conductivity. The tangential component of electric field $\hat{n} \times \hat{n} \times \vec{E}$ and the normal components of magnetic field $(\hat{n} \cdot \vec{H})$ must be both are zero on s ($\hat{n}$ is the unit vector normal to the surface 's').

Now, expand $\vec{E} and\ \vec{H}$ in terms of orthogonal set of basis vector field $E_a\ and\ \vec{H}_a$, respectively.

Here we consider two relations-

$$\boldsymbol{k_a \vec{E}_a = \vec{\nabla} \times \vec{H}_a} \qquad \textbf{..... (1)}$$

$$\text{And } \boldsymbol{\vec{\nabla} \times \vec{E}_a = k_a \vec{H}_a} \qquad \textbf{.... (2)}$$

Where k_a is a constant to be determined

The above two relations are called **slater decomposition.**

Now we take curl on both side of equation (1)-

$$k_a(\vec{\nabla} \times \vec{E}_a) = \vec{\nabla} \times (\vec{\nabla} \times \vec{H}_a)$$

$$k_a(k_a \vec{H}_a) = \vec{\nabla}(\vec{\nabla} \cdot \vec{H}_a) - \nabla^2 \vec{H}_a$$

$$\nabla^2 \vec{H}_a + {k_a}^2 H_a = 0 \qquad ..... (3)$$

Similarly,

$$\nabla^2 \vec{E}_a + {k_a}^2 \vec{E}_a = 0 \qquad ..... (4)$$

The equation (3) and (4) are the well known wave equation.

Now, We consider a closed contour 'c' on 's' surrounding a surface $\acute{s}$.

$$\oint \vec{E}_a \cdot \vec{dl} = \oint (\hat{n} \times \hat{n} \times \vec{E_a}).\vec{dl} + \oint (\hat{n}.\vec{E}_a)\hat{n}.\vec{dl} = 0$$

Tangential component is zero.

$$\oint \vec{E}_a.\vec{dl} = \oint (\hat{n}.\vec{E}_a)\hat{n}.\vec{dl} = \int (\vec{\nabla} \times \vec{E}_a).\hat{n}da = \int k_a(\vec{H}_a.\hat{n})da = k_a \int (\vec{H}_a.\hat{n})da = 0$$

$$\boxed{\int (\overrightarrow{H_a}.\hat{n})da = 0}$$

Here, $\vec{E}_a\ and\ \vec{E}_b$ are orthogonal to each other i.e,

$\int \vec{E}_a.\vec{E}_b$ dv = 0 (a$\neq b$) …...(5)

$\int \vec{H}_a.\vec{H}_b$ dv = 0 (a$\neq b$) …… (6)

In order to prove the orthogonality condition

$\vec{\nabla}.(\vec{A}\times\vec{B}) = \vec{B}.(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{B})$

To $(\vec{E}_b \times \vec{\nabla} \times \vec{E}_a)$ and then to $(\vec{E}_a \times \vec{\nabla} \times \vec{E}_b)$,and subtract them .

Then we have –

$\vec{\nabla}.(\vec{E}_b \times \vec{\nabla} \times \vec{E}_a)$- $\vec{\nabla}.(\vec{E}_a \times \vec{\nabla} \times \vec{E}_b)$=$(\vec{\nabla}\times\vec{E}_a).(\vec{\nabla}\times\vec{E}_b) - \vec{E}_b\{\vec{\nabla}\times(\vec{\nabla}\times\vec{E}_a)\} - (\vec{\nabla}\times\vec{E}_a).(\vec{\nabla}\times$ $\vec{E}_{sb}) + \vec{E}_a\{\vec{\nabla}\times(\vec{\nabla}\times\vec{E}_b)\}$

=$\vec{E}_a.\{\vec{\nabla}\times(\vec{\nabla}\times\vec{E}_b)\}$-$\vec{E}_b\{\vec{\nabla}\times(\vec{\nabla}\times\vec{E}_a)\}$

$\int \vec{\nabla}.(\vec{E}_b \times \vec{\nabla} \times \vec{E}_a) - \vec{\nabla}.(\vec{E}_a \times \vec{\nabla} \times \vec{E}_b)$dv = $\int \vec{E}_a.\{\vec{\nabla}\times(\vec{\nabla}\times\vec{E}_b)\} - \vec{E}_b\{\vec{\nabla}\times(\vec{\nabla}\times\vec{E}_a)\}$ dv

$\int[k_a\vec{\nabla}.(E_b \times \vec{H}_a) - k_b\vec{\nabla}.(\vec{E}_a \times \vec{H}_b)]dv = \int \vec{E}_a.\{\vec{\nabla}\times(\vec{\nabla}\times\vec{E}_b)\} - \vec{E}_b\{\vec{\nabla}\times(\vec{\nabla}\times\vec{E}_a)\}$ dv

$\vec{k}_a\int \hat{n}.(\vec{E}_b \times \vec{H}_a)ds - \vec{k}_b\int \hat{n}.(\vec{E}_a \times \vec{H}_b)ds = \int \vec{E}_a.\{\vec{\nabla}\times(\vec{\nabla}\times\vec{E}_b)\} - \vec{E}_b\{\vec{\nabla}\times(\vec{\nabla}\times\vec{E}_a)\}$ dv

$=({k_b}^2 - {k_a}^2)\int(\overrightarrow{E_a}.\vec{E}_b)dv$ ...... (7)

Where $\vec{\nabla}\times\vec{E}_a = k_a\vec{H}_a \; and \; \vec{\nabla}\times\vec{\nabla}\times\vec{E}_a = k_a(\vec{\nabla}\times\vec{H}_a) = {k_a}^2\vec{E}_a$

Applying divergence theorem, LHS=0. RHS of equation (7) becomes,

$({k_b}^2 - {k_a}^2)\int \vec{E}_a.\vec{E}_b dv = 0$ …… (8)

Here $k_a \neq k_b$

Thus,$\int \vec{E}_a.\vec{E}_b = 0$

If $k_a = k_b$ $i.e$, when $\vec{E}_a \; and \; \vec{E}_b$ are the numbers of a degenerate basis set, it is possible to construct linear supper-position of the degenerate functions so that orthogonality preserved.

Now, $\int \vec{E}_a.\vec{E}_b \, dv = \delta ab$

Similarly,

$$\int \vec{H}_a.\vec{H}_b \, dv = \delta ab$$

The total resonator fields of $\vec{E}(r,t)$ and $\vec{H}(r,t)$ can be expanded as-

$\vec{E}(r,t) = -\sum_a \frac{1}{\sqrt{\epsilon}} p_a(t)\vec{E}_a(r)$ …. (9)

$\vec{H}(r,t) = \sum_a \frac{1}{\sqrt{\mu}} \omega_a q_a(t)\vec{H}_a(r)$ ….. (10)

Where $\omega_a = \frac{k_a}{\sqrt{\mu\epsilon}}$

Now , we know

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$-\sum_a \frac{1}{\sqrt{\epsilon}} p_a(t) k_a H_a = -\sqrt{\mu}\sum_a \omega_a \dot{q}_a(t)\vec{H}_a$$

Comparing we get –

$$\frac{k_a p_a}{\sqrt{\mu\,\epsilon}} = \omega_a \dot{q}_a$$

$\dot{q}_a = p_a$

Using fourth Maxwell equation ($\vec{\nabla} \times \vec{H} = \mu \in \frac{\partial \vec{E}}{\partial t}$)-

$\dot{p}_a = -{\omega_a}^2 q_a$

$\ddot{p}_a = -{\omega_a}^2 \dot{q}_a = -{\omega_a}^2 p_a$

$\ddot{p}_a + {\omega_a}^2 p_a = 0$ ……. (11)

3. Photon number distribution:

We are always interested to know about the photon statistics in various radiation field so that we can understand the properties of that field. Here we consider two type of photon number distribution one is for thermal field which is classical in nature and another is for coherent state which is treated as quantum mechanically.

3.1. For thermal field

According to the statistical mechanics P_n is the probability of the mode in thermally excited state. Hence we have

$$P_n = \frac{e^{\frac{-E_n}{kT}}}{\sum e^{\frac{-E_n}{kT}}} \qquad \text{…. (1)}$$

[Where k=Boltzmann constant]

The denominator is a usual partition function and it given by

$$Z = \sum_N e^{\frac{-E_n}{kT}}$$

Assuming that the electromagnetic field behaves as Harmonic oscillator, the energy of nth state is described as

$$E_n = (n + 1/2)\hbar\omega \qquad \text{n = number of energy state.}$$

Substituting the value of E_n in equation (2) we get

$$Z = \sum_N e^{\frac{-E_n}{kT}}$$

$$\text{or } Z=\sum e^{-\left(n+\frac{1}{2}\right)\hbar\omega}$$

$$\text{or } Z=\sum e^{-\frac{\left(\frac{1}{2}\right)\hbar\omega}{kT}} \sum e^{-\frac{n\hbar\omega}{kT}}$$

$$\text{Let } \frac{\hbar\omega}{kT} = x$$

$$\sum e^{-n\frac{\hbar\omega}{kT}} = \sum e^{-nx}$$

$$= 1 + e^{-x} + e^{-2x}$$

$$= \frac{1}{1 - e^{-x}}$$

$Z=e^{-\left(\frac{1}{2}\right)x} \frac{1}{1-e^{-x}}$ …… (3)

Now the average photon number can be calculated as-

$$\bar{n} = \sum nP_n$$

$$= \frac{\sum ne^{-E_n/kT}}{\sum e^{-E_n/kT}}$$

$$= \frac{1}{Z} \sum ne^{-(n+1/2)\hbar\omega}$$

$$= \frac{1}{Z} e^{-x}/_2 \sum ne^{-nx}$$

$\bar{n} = \frac{1}{Z} e^{-x}/_2 \frac{e^{-x}}{(1-e^{-x})^2}$ ….. (4)

Substituting the value of Z in equation (4) we get, $\bar{n} = \frac{1}{e^x - 1}$

Now, $\frac{\bar{n}^n}{(1+\bar{n})^{n+1}} = \frac{(e^x-1)^n}{e^{(n+1)x}} \frac{1}{(e^x-1)^n}$

$$= (1-e^{-x})e^{-nx} \ldots \qquad \ldots..(5)$$

Again from expression (1) we can write

$$p_n = \frac{e^{\frac{-E_n}{kT}}}{\sum e^{\frac{-E_n}{kT}}}$$

$$\text{Or, } P_n = \frac{e^{-\left(n+\frac{1}{2}\right)\hbar\omega/kT}}{\sum e^{-\left(n+\frac{1}{2}\right)\hbar\omega/kT}}$$

$$= \frac{e^{-x/2}\, e^{-nx}}{e^{-x/2} \sum e^{-nx}}$$

$$= \frac{e^{-nx}}{\sum e^{-nx}} \qquad \ldots\ldots (6)$$

$$P_n = \frac{e^{-nx}}{\frac{1}{(1-e^{-x})}}$$

$$\text{Or, } P_n = (1 - e^{-x})e^{-nx} \qquad \ldots\ldots (7)$$

From equation (5) and (7) we can write

$$P_n = \frac{\overline{n^n}}{(1+\bar{n})^{n+1}}$$

3.2. Photon number distribution for coherent state or laser field:

A coherent state in quantum mechanics is a specific quantum state of the quantum harmonic oscillator, characterized by its dynamics closely resembling the oscillatory behavior of a classical harmonic oscillator. Erwin Schrödinger first derived this state in 1926 while seeking solutions to the Schrödinger equation that align with the correspondence principle. Coherent states appear in the quantum theory of many physical systems, such as describing the oscillatory motion of a particle in a quadratic potential well. In such systems, the coherent state represents a ground-state wave packet displaced from the system's origin.

Mathematically, a coherent state is defined to be the (unique) eigenstate of the annihilation operator $\hat{a}$ associated to the eigenvalue α. Formally, this reads,

$$\hat{a}\ |\alpha\rangle = \alpha\ |\alpha\rangle$$

Since $\hat{a}$ is not hermitian α is, in general, a complex number.

Physically, this formula means that a coherent state remains unchanged by the annihilation of field excitation or, say, a particle. An eigen state of the annihilation operator has a Poissonian number distribution when expressed in a basis of energy eigen states, as shown belong to Poissonian distribution is a necessary and sufficient condition that all detections are statistically independent. Compare this to a single-particle state: once one particle is detected, there is zero probability of detecting another.

The derivation of this will make use of *dimensionless operators*, X and P, normally called *field quadratures* in quantum optics. These operators are related to the position and momentum operators of a mass m on a spring with constant k,

$$P = \sqrt{\frac{1}{2\hbar m\omega}}\,\hat{p}$$

$$X = \sqrt{\frac{m\omega}{2\hbar}}\,\hat{x} \qquad \text{where } \omega = \sqrt{\frac{k}{m}}$$

We define a eigen vector $|\alpha\rangle$ of a non Hermitian operator 'a' . Now , we expand $|\,\alpha\rangle$

$$|\,\alpha\rangle = \sum_{n=0}^{\alpha} |n\rangle\langle n\,|\alpha\rangle$$

$$= \sum_{n=0}^{\alpha} C_n(\alpha)\ |n\rangle$$

Where $C_n(\alpha) = \langle n\ |\alpha\rangle$ is the transformation between the number and coherent state representation. $|\langle n\ |\alpha\rangle\ |^2$ represents the probability of finding the oscillation in the state $|\alpha\rangle$.

$$a\ |\alpha\rangle = \sum_{n=1}^{\alpha} C_n(\alpha)\sqrt{n}\ |n-1\rangle$$

$$= \sum_{n=0}^{\alpha} \alpha\, C_n(\alpha)\ |n\rangle$$

The first sum runs from 1 to α because the n=0 term gives zero. Therefore we may shift indices and n replaced by (n+1), so this becomes,

$$\sum_{n=0}^{\alpha} C_{n+1}(\alpha)\sqrt{n+1}\ |n\rangle = \sum_{n=0}^{\alpha} \alpha C_n(\alpha)\ |n\rangle$$

Now, we multiply both side by ⟨m|

Here, ⟨m |n⟩=δ_{mn}

We get, $C_{n+1}(\alpha)\sqrt{n+1} = \alpha C_n(\alpha)$

For, n=0 $C_1 = \frac{\alpha}{\sqrt{1}} C_0$

n=1 $C_2 = \frac{\alpha}{\sqrt{2}} C_1 = \frac{\alpha^2}{\sqrt{2!}} C_0$

n=2 $C_3 = \frac{\alpha}{\sqrt{3}} C_2 = \frac{\alpha^3}{\sqrt{3!}} C_0$

$C_n(\alpha) = \frac{\alpha^n}{\sqrt{n!}} C_0$

Therefore, we can write

$|\alpha\rangle = C_0 \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\ |n\rangle$

We choose C_0, such that

$\langle\alpha\ |\alpha\rangle = 1 = \ |C_0\ |^2 \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{\alpha^{*m}\alpha^n}{\sqrt{n!m!}} \langle m\ |n\rangle$

Put (n=m) $= |C_0\ |^2 \sum_{n=0}^{\infty} \frac{(\ |\alpha\ |^2)^n}{n!}$

$= |C_0\ |^2 e^{(\ |\alpha\ |^2)}$

⟨n $|\alpha\rangle = C_n(\alpha) = e^{-\ |\alpha\ |^2/2}\ \frac{\alpha^n}{\sqrt{n!}}$

Thus, $P_n(\alpha) = \ |\langle n\ |\alpha\rangle\ |^2 = e^{-\ |\alpha\ |^2}\ \frac{\alpha^{2n}}{n!}$

This is the photon number distribution in coherent state.

Therefore, we can infer from the coherent state and thermal field photon number distributions that the coherent state photon number distribution preserves poissonian statistics while the thermal field does not.

4. Non-classical light.

Non-classical light is light that cannot be described using classical electromagnetism; its characteristics are described by the quantized electromagnetic field and quantum mechanics.

We have emphasized the fact that all states of light are quantum mechanical and are thus non-classical, deriving some quantum features from the discreteness of the photons in the previous part. Of course, in practice, the non-classical features of light are difficult to observe. (We shall use “quantum mechanical” and “non-classical” more or less interchangeably here.) As we shall see, it is possible to have non-classical states involving a very large number of photons. But we need a criterion for non-classicality. We can understand this in terms of density matrix, in Glauber-Sudarshan P representation.

The **Glauber–Sudarshan P representation** is a suggested way of writing down the phase space distribution of a quantum system in the phase space formulation of quantum mechanics. The P representation is the quasi-probability distribution in which observables are expressed in normal order. In quantum optics, this representation is sometimes championed over alternative representations to describe light in optical phase space, because typical optical observables, such as the particle number operator, are naturally expressed in normal order. It is named after George Sudarshan and Roy J. Glauber who were working on the topic in 1963.

The density matrix for any state of light can be written as:

$\hat{\rho} = \int P(\alpha)\ |\alpha\rangle\langle\alpha|\ d^2\alpha$

Where $|\alpha\rangle$ Is a coherent state. The quasi-probability P(α) is said to be non-classical if P(α) has a negative value at any point.

The States for which P(α) is positive everywhere or no more singular than a delta function, are classical whereas those for which P(α) is negative or more singular than a delta function are non-classical. P(α) for a coherent state is a delta function, and Hillery has shown that all other pure states of the field will have functions P(α) that are negative in some regions of phase space and are more singular than a delta. It is evident that the variety of possible non-classical states of the field is quite large. Some of the most significant instances of non-classical states will be covered in this chapter. We start by talking about the squeezed states, which include the number and quadrature squeezed states, before going over the non-classical nature of photon antibunching once more.

4.1. Squeezed States:

States for which P(α) is positive or no more singular than a delta function are, in this sense, classical. Coherent states, which we have already shown to be quasi-classical in that they describe states of the field having properties close to what we would expect for classical oscillating coherent fields, and they have P functions given as delta functions, and therefore classical in the sense described here. Certain effects, among them being quadrature and amplitude squeezing, can occur only for states for which the P functions are negative or highly singular. For that reason, the various forms of squeezing are known as distinctly non-classical effects.

If two operators $\hat{A}$ and $\hat{B}$ satisfy the commutation relation $[\hat{A},\hat{B}]$ = iĈ,

It follows that,

$\langle(\Delta\hat{A})^2\rangle\langle(\Delta\hat{B})^2\rangle \geq \frac{1}{4}|\langle\hat{C}\rangle|^2$ …..(1)

A state of the system is said to be squeezed if either

$\langle(\Delta\hat{A})^2\rangle < \frac{1}{2}|\langle\hat{C}\rangle|$

Or $\langle(\Delta\hat{B})^2\rangle < \frac{1}{2}|\langle\hat{C}\rangle|$ …….. (2)

(Because of Eq. (1), we obviously cannot have both variances less than $|\langle\hat{C}\rangle|/2$ simultaneously.) Squeezed states

for which the $[\widehat{X_1},\widehat{X_2}]$=i/2 equality holds in Eq. (1) are sometimes known as ideal squeezed states and are an example of the "intelligent" states . In the case of quadrature squeezing, we take

$\hat{A} = \widehat{X_1}$ and

$\hat{B} = \widehat{X_2}$

With $\widehat{X_1}$ *and* $\widehat{X_2}$ being the quadrature operators of Eqs.

$$\widehat{X_1} = \frac{1}{2}(\hat{a} + \widehat{a^\dagger})$$

$$\widehat{X_2} = \frac{1}{2}(\hat{a} - \widehat{a^\dagger})$$

Satisfying the Eq . $[\widehat{X_1},\widehat{X_2}] = \frac{i}{2}$ ……. (3)

and thus $\hat{C}$ =1/2.

From the

Eq. $\langle(\Delta\widehat{X_1})^2\rangle\langle(\Delta\widehat{X_2})^2\rangle \geq \frac{1}{16}$

It follows that quadrature squeezing exists whenever

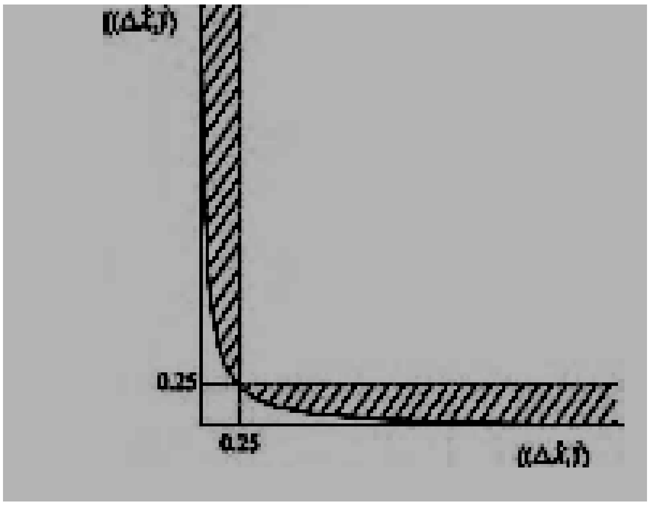
$\langle(\Delta\widehat{X_1})^2\rangle$<1/4 $\langle(\Delta\widehat{X_2})^2\rangle$<1/4 …….(4) .

We have already established that, for a coherent state |α⟩, the equality in Eq.(3) holds and that the variances of two quadratures are equal

$\langle(\Delta\widehat{X_1})^2\rangle = \langle(\Delta\widehat{X_2})^2\rangle$=1/4

Not only that, but the result for the coherent state is exactly the same as for the vacuum . States for which one of t

He conditions in Eq. (4) holds will have less "noise" in one of the quadratures than for a coherent state or a vacuum state – the fluctuation in that quadrature are squeezed. Of course, the fluctuation in the other quadrature must be enhanced so as to not violate the uncertainty relation. There are squeezed states for which the uncertainty relation is equalized, but this need not be the case in general.



The figure is for a graphical representation of the range of squeezing.

Before presenting specific examples of squeezed states, we wish to demonstrate why quadrature squeezing must be considered an on classical effect. To this end, we express the relevant expectation values in terms of the P function.

$\langle(\Delta\widehat{X_1})^2\rangle=\frac{1}{4}\{1+\int P(\alpha)\,[(\alpha+\alpha^*)-(\langle\hat{a}\rangle+\langle\hat{a}^\dagger\rangle)]^2 d^2\alpha\}$

$\langle(\Delta\widehat{X_2})^2\rangle=\frac{1}{4}\{1+\int P(\alpha)\,[(\alpha-\alpha^*)/i-(\langle\hat{a}\rangle-\langle\hat{a}^\dagger\rangle)/i]^2 d^2\alpha\}$

Where

$\langle\hat{a}\rangle=\int P(\alpha)\alpha d^2\alpha$ and $\langle\hat{a}^\dagger\rangle=\int P(\alpha)\alpha^* d^2\alpha$

Because the term inside the square brackets is, of course, always positive, it is evident that the condition

$\langle(\Delta\hat{X}_{1,2})^2\rangle< 1/4$ requires that P(α) be non positive at least in some regions of phase space.

It is sometimes convenient to introduce a generic quadrature operator

$\hat{X}(\vartheta)=\frac{1}{2}(\hat{a}e^{-i\vartheta}+\hat{a}^\dagger e^{-i\vartheta})$

Where obviously $\hat{X}(0)=\widehat{X_1}$ and $\hat{X}\left(\frac{\pi}{2}\right)=\widehat{X_2}$.

To characterizing squeezing, we introduce the parameter

S $(\vartheta)=\frac{\langle\nabla\hat{X}(\vartheta)^2\rangle-1/4}{1/4}$=4 $\langle(\Delta\hat{X}(\vartheta))^2\rangle$-1

Squeezing exists when angle ϑ will be -1$\leq s(\vartheta)<0$.

For more information about "squeeze" we define a operator

$\hat{s}(\xi)=\exp[\frac{1}{2}(\xi^* a^2-\xi {a^\dagger}^2)]$ …… (5)

Where **ξ**=r$e^{i\theta}$,

And where r is known as the squeeze parameter and 0≤ r < ∞ and 0≤ θ ≤ 2π.This operator $\hat{s}(\xi)$is a kind of two-photon generalization of the displacement operator used to define the usual coherent states of a single-mode field. Evidently, the operator $\hat{s}(\xi)$ acting on the vacuum would create some sort of "two-photon coherent state" as It is clear that photons will be created or destroyed in pairs by the action of this operator

.4.2. Photon anti-bunching

To study photon anti-bunching first we have to understand Coherence function. Coherence is defined as the ability of waves to interfere. Coherent waves have a well-defined constant phase relationship. Coherence functions, introduced by Glauber and others in the 1960s, define the correlation between electric field components, capturing the mathematical basis of coherence. These correlations can be measured at different orders, leading to the concept of various coherence levels. First-order coherence, seen in experiments like Young's double slit and the Mach-Zender interferometer, relates to field correlations. In 1956, Hanbury Brown and Twiss introduced second-order coherence, which focuses on intensity correlations. Higher-order coherence becomes important in photon-coincidence counting. While classical and quantum coherence functions may give similar results, the quantum description differs, dealing with interfe rence of possible histories rather than physical waves.

The electric field $E(r,t)$ can be separated into its positive and negative frequency components $E(r,t) = E^{+}(r,t) + E^{-}(r,t)$.

Either of the two frequency components, contains all the physical information about the wave. The classical first-order, second order and n-th order correlation function are defined as follows

$$G_C{}^{(1)}(x_1, x_2) = \langle E^{-}(x_1)E^{+}(x_2)\rangle,$$

$$G_c{}^{(2)}(x_1, x_2) = \langle E^{-}(\mathrm{x}_1)E^{-}(\mathrm{x}_2)E^{+}(\mathrm{x}_3)E^{+}(\mathrm{x}_4)$$

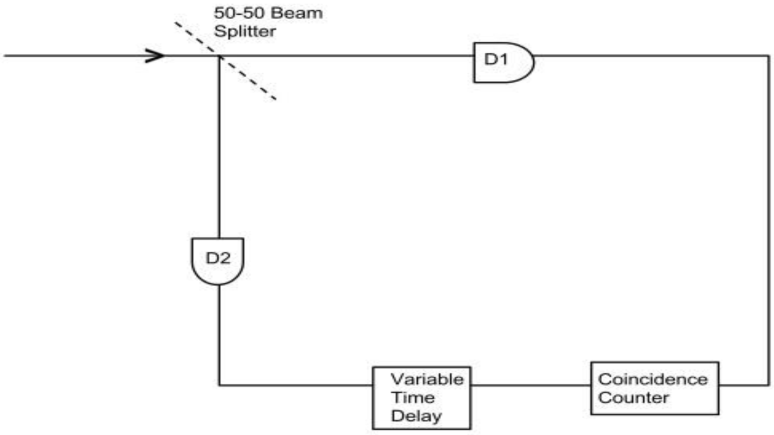
$$G_C{}^{(n)}(x_1, x_2, \ldots., x_n) = \langle E^{-}(x_1) \ldots \ldots E^{-}(x_n)E^{+}(x_{n+1})E^{+}(x_{2n})\rangle,$$

Where x_i represents (r_i, t_i). While the order of the$E^{+}(r,t)$ and$E^{-}(r,t)$, does not matter in the classical case, as they are merely numbers and hence commute, the ordering is vital in the quantum analogue of these correlation functions.

4.3. First order coherence function: First order coherence understood mathematically by Young's double slit experiment. .Such an experiment is able to determine the degree to which a light source is monochromatic, or to determine the coherence length of the light, but it says nothing about the statistical properties of the light. That is, first-order coherence experiments are unable to distinguish between states of light with identical spectral distributions but with quite different photon number distributions.

4.4. Higher order Coherence Function

Brown and Twiss experiment: In the 1950s, Hanbury Brown and Twiss in Manchester developed a new kind of correlation experiment that involved the correlation of intensities rather than of fields. A sketch of the experiment is shown in Fig. bellow. Detectors D1 and D2 are at same distance from the beam splitter. This setup measures a delayed coincidence rate where one of the detectors registers a count at time t and the other a count at t +τ. If τ, the time delay, is smaller than the coherence time τ_0, information on the statistics of the light beam striking the beam splitter can be determined.



Above Fig. shows the experimental setup of Brown and Twiss experiment.

Classical theory:

The rate of coincident counts is proportional to the time, or ensemble, average

C (t, t +τ) = ⟨I (t) I (t +τ)⟩ …..(1)

Where I(t) and I(t +τ) are the instantaneous intensities at the two detectors (these are classical quantities here). If we assume that the fields are stationary, the average is a function only of t. If the average of the intensity at each detector is ⟨I(t)⟩, then the probability of obtaining a coincidence count with time delay τ is

$\gamma^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau)\rangle}{\langle I(t)\rangle^2}$ …… (2)

$= \frac{\langle E^*(t)E^*(t+\tau)E(t+\tau)E(t)\rangle}{\langle E^*(t)E(t)\rangle^2}$ ……. (3)

This is the classical second-order coherence function. If the detectors are at different distances from the beam splitter, the second-order coherence function can be generalized to

$\gamma^{(2)}\ (x_1, x_2\ ;\ x_1, x_2) = \frac{\langle I(x_1)I(x_2)\rangle}{\langle I(x_1)\rangle\langle I(x_2)\rangle}$

$= \frac{\langle E^*(x_1)E^*(x_2)E(x_2)E(x_1)\rangle}{\langle |E(x_1)|^2\rangle\langle |E(x_1)|^2\rangle}$ …….. (4)

By analogy to first-order coherence, there is said to be classical coherence to second order if $|\gamma^{(1)}(x_1, x_2)|$=1 and $\gamma^{(2)}(x_1, x_2; x_1, x_2)$ =1.

The second condition requires the factorization

$\langle E^*(x_1)E^*(x_2)E(x_2)E(x_1)\rangle$=. $\langle |E(x_1)|^2\rangle\langle |E(x_1)|^2\rangle$ …….. (5)

Consider a plane wave propagating in the z-direction, the equation is given by

E(z,t+τ)=$E_0 e^{i[kz]-\omega(t+\tau)]}$

From (4) it is easy to show that

$\langle E^*(t)E^*(t+\tau)$E(t+$\tau$)E(t)⟩=$E_0{}^4$ ………..(6)

and thus $\gamma^{(2)}$(τ)=1. ...…… (7)

For any light beam of constant, non-fluctuating, intensity, we have

I(t)= I(t +τ)= I_0,

Thus $\gamma^{(2)}$(τ)=1.

However, the second-order coherence function is not restricted to be unity or less. To see this, we first consider the zero time-delay coherence function

Which is given by →

$$\gamma^{(2)}(0) = \frac{\langle I(t)^2\rangle}{\langle I(t)\rangle^2}$$

For a sequence of N measurements taken at times $t_1, t_2 \ldots.. t_N$, the required averages are given by

⟨I(t)⟩=$\frac{I(t_1)+I(t_2)+\ \ldots\ +I(t_N)}{N}$ ………..(8)

$\langle I(t)^2\rangle = \frac{I(t_1)^2+I(t_2)^2+\cdots+I(t_N)^2}{N}$ …….. (9)

Now according to Cauchy's inequality applied to a pair of measurements at time $t_1 and\ t_2$, we have

2I (t_1)I(t_2)<=$I(t_1)^2 I(t_2)^2$ ……..(10)

Applying this to all the cross terms in $< I(t)^2 >$ it follows that

⟨I(t)^2⟩ ≥ $\langle I(t)\rangle^2$. …….. (11)

And thus

1<=$\gamma^{(2)}(0)$<∞ ……… (12)

∴ There being no way to establish an upper limit.

For nonzero time delays, the positivity of the intensity ensures that

1<=$\gamma^{(2)}$(τ)<∞ $(\tau \neq 0)$ ……………….(13)

But from the inequality equation (11) it can be established that

$$[I(t_1)I(t_1+\tau)+\ \ldots.+I(\mathrm{t_N}+\tau)]^2 \le [I(t_1)^2+\ \ldots.I(t_N)^2][\mathrm{I}(\mathrm{t_1}+\tau)^2+\ \ldots.+\mathrm{I}(\mathrm{t_N}+\tau)^2]$$

…… (13)

For a long series for many measurements, the two series on the R.H.S are equivalent so that

⟨I(t)I(t+τ)⟩≤ $\langle I(t)\rangle^2$

Thus we arrive at

$\gamma^2(\tau) \leq \gamma^2(0)$ …….(14)

The results reported in Eqs. (11) and (14) establish limits for classical light fields. Later we shall show that some quantum states of light violate the quantum mechanical version of the inequality of Eq. (14). For a light source containing a large number of independently radiating atoms undergoing collisional broadening, it can be shown that the first and second order coherence functions are related according to

$\gamma^{(2)}(\tau)=1+|\gamma^{(1)}(\tau)|^2$ …….. (15)

This is a relation that holds for all kinds of chaotic light. Evidently, since $0\leq|\gamma^{(1)}(\tau)|\leq1$ it follows that $1\leq|\gamma^{(2)}(\tau)|\leq2$.

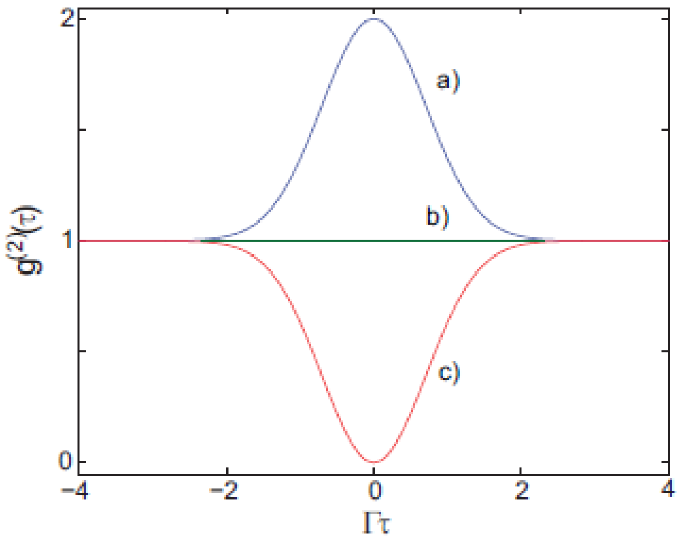
From the result in the Equation $\gamma^{(1)}(\tau) = e^{-i\omega_0\tau-|\tau|/\tau_0}$ ……… (16)

$$where \;\; 0 \leq \gamma^{(1)}(\tau) \leq 1$$

we have, this gives Lorentzian spectra,

$\gamma^{(2)}(\tau) = 1 + e^{-2|\tau|/\tau_0},$ …………. (17)

Although for $\tau \to\infty$, $\gamma^{(2)}(\tau)\to1$, and for zero time delay, $\tau \to 0$, $\gamma^{(2)}(0)=2$. In fact, for any kind of chaotic light, $\gamma^{(2)}(0)=2$.The implication of this result is as follows. If light incident on one of the detectors is independent of the light incident on the other, there should be a uniform coincidence rate independent of t. This is what Hanbury Brown and Twiss expected. By using an elementary (but wrong!) picture in which the photons are emitted independently by the source, and assuming that the beam splitter did not split photons but merely reflected or transmitted them, Hanbury Brown and Twiss expected to be able to demonstrate the existence of photons. They found, for zero time delay, twice the detection rate compared with the rate at long time delays. If photons exist, they evidently arrive in pairs at zero time delay but independently at long time delays. That photons arrive in "bunched" pairs is now known as the **photon bunching effect** (also known as the Hanbury Brown and Twiss effect). Note that by measuring the coincidence counts at increasing delay times, it is possible to measure the coherence time τ_0 of the source.



Sketch of a second order correlation function for a) classical light with Gaussian frequency distribution. b) coherent light and c)non-classical light emitted by an ion(bottom).

Quantum theory:

In the previous paragraph, we spoke of the bunching of photons, even though the important result of Eq. (16) was not derived on the basis of the quantum theory of light. We now introduce the second-order quantum coherence function and show that light in a coherent state,as obtained (to a reasonable approximation) from a well-stabilized laser, is coherent to second order and that thermal light sources exhibit the photon bunching effect. In these cases, the quantum and classical pictures agree, but it will become clear that there are instances where the quantum theory predicts situations for which there is no classical counterpart.

The transition probability for the absorption of two photons is proportional to

$$|\langle f|\hat{E}^{(+)}(r_2,t_2)\hat{E}^{(+)}(r_1,t_1)|i\rangle|^2 \qquad …….. (18)$$

Which after summation over all final states becomes

$$\langle i|\hat{E}^{(-)}(r_2,t_2)\hat{E}^{(-)}(r_1,t_1)\hat{E}^{(+)}(r_2,t_2)\hat{E}^{(+)}(r_1,t_1)|i\rangle$$

Generalizing to cases of non pure field states we introduce the second –order quantum correlation function

$$G^{(2)}(x_1,x_2;x_2,x_1) = Tr\{\hat{\rho}\hat{E}^{(-)}(x_1)\hat{E}^{(-)}(x_2)\hat{E}^{(+)}(x_1)\hat{E}^{(+)}(x_2)\} \quad \text{…….. (19)}$$

which is to be interpreted as the ensemble average of I(x_1)I(x_2). As in the first order case, the normal ordering of the field operators for absorptive detection is important and must be preserved. We define the second-order quantum coherence function as

$$g^{(2)}(x_1,x_2;x_2,x_1) = \frac{G^{(2)}(x_1,x_2;x_2,x_1)}{G^{(1)}(x_1,x_1)G^{(1)}(x_2,x_2)} \quad \text{……… (20)}$$

Where $g^{(2)}(x_1,x_2;x_2,x_1)$ is the joint probability of detecting one photon at r_1 *at the time* t_1 and a second

r_2 *at the time* t_2 . A quantum field is said to be second order coherent if $|g^{(1)}(x_1,x_2)|$ =1and

$g^{(2)}(x_1,x_2;x_2,x_1) = 1$. This requires that $G^{(2)}(x_1,x_2;x_2,x_1)$factorized according to

$$G^{(2)}(x_1,x_2;x_2,x_1) = G^{(1)}(x_1,x_1)G^{(1)}(x_2,x_2) \quad \text{…….. (21)}$$

At a fixed position $g^{(2)}$ depends only on the time of difference $\tau = t_2 - t_1$

$$g^{(2)}(\tau) = \frac{\langle\hat{E}^{(-)}(t)\hat{E}^{(-)}(t+\tau)\hat{E}^{(+)}(t)\hat{E}^{(+)}(t+\tau)\rangle}{\langle\hat{E}^{(-)}(t)\hat{E}^{(+)}(t)\rangle\langle\hat{E}^{(-)}(t+\tau)\hat{E}^{(+)}(t+\tau)\rangle} \quad \text{……… (22)}$$

Which is interpreted as the conditional probability that if a photon is detected

at t another is also detected at t+τ .

For a single mode field, $g^{(2)}(\tau)$ reduce to

$$, \; g^{(2)}(\tau) = \frac{\langle\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}\rangle}{\langle\hat{a}^\dagger\hat{a}\rangle^2}$$

$$= \frac{\langle\hat{n}(\hat{n}-1)\rangle}{\langle\hat{n}\rangle^2}$$

$$= 1 + \frac{\langle(\Delta\hat{n})^2\rangle - \langle\hat{n}\rangle}{\langle\hat{n}\rangle^2} \quad \text{……. (23)}$$

Which is independent of τ.

For the field in a coherent state $|\alpha\rangle$it follows that

$$g^{(2)}(\tau)=1 \quad \text{…….. (24)}$$

it means that the probability of a delayed coincidence is independent of time. This state is second-order coherent.

For a field in a single-mode thermal state(all other modes filtered out) it can be shown that

$$g^{(2)}(\tau)=2 \quad \text{……… (25)}$$

indicating a higher probability of detecting coincident photons. For a multimode (unfiltered) thermal state it can be shown that, just as in the classical case,

$$g^{(2)}(\tau)=1+|g^{(1)}(\tau)|^2 \quad \text{……….(26)}$$

which lies in the range 1≤ $g^{(2)}(\tau)$≤ 2. For collision broadened light with a Lorentzian spectrum and a first-order coherence function

$$g^{(1)}(\tau) = e^{-i\omega_0\tau - |\tau|/\tau_0} \quad \text{……….(27)}$$

we have

$$g^{(2)}(\tau)=1+e^{-2|\tau|/\tau_0}$$

which is just as in the classical case. For $|\tau| \ll \tau_0$, the probability of getting two photon counts within the time |τ| is large compared with the random case. For zero time delay, $g^{(2)}(0)=2$, and $g^{(2)}(\tau) < g^{(2)}(0)$. This inequality characterizes photon bunching. For a multimode coherent state, using the definition of Eq. (22), it can be shown that $g^{(2)}(\tau)=1$.

And thus the photons arrive randomly as per the Poisson distribution, $g^{(2)}(\tau)$ being independent of the delay time.

But there is another possibility, the case where $g^{(2)}(0) < g^{(2)}(\tau)$. This is the opposite of photon bunching, photon antibunching.

For this, the photons tend to arrive evenly spaced in time, the probability of obtaining coincident photons in a time interval τ is less than for a coherent state (the random case).,this situation is quite non classical in the sense that apparent negative probabilities are involved, meaningless for classical fields. But for now let us consider the single-mode field in a number state |n⟩ from which it follows that

$g^{(2)}(\tau) = g^{(2)}(0) = 0$ for n =0,1

$=1-\frac{1}{n}$ for n ≥2 …………. (28)

Evidently $g^{(2)}(0) < 1$ and this is outside the allowed range for its classical counter .The fact that $g^{(2)}(0)$ taken as classically forbidden values may be interpreted as a quantum-mechanical violation of the Cauchy inequality. $g^{(2)}(0)$ will be less than unity whenever $\langle(\Delta n)^2\rangle < \langle\hat{n}\rangle$. States for which this condition holds are *sub-Poissonian*. Since $g^{(2)}(\tau)$ is constant for the single-mode field, photon antibunching does not occur, the requirement for it to occur being

$g^{(2)}(0) < g^{(2)}(\tau)$. *The point is that photon anti bunching and sub-Poissonian statistics are different effects although they have often been confused as being essentially the same thing.* We will discuss this later.

In the previous Section we discussed the second-order coherence function $g^{(2)}(\tau)$ of Eq. (21) characterizing the joint probability of detecting one photon followed by another, within the delay time τ at some fixed position. If $g^{(2)}(\tau)=1$ then the photons arrive independently, and, in fact, we should expect that $g^{(2)}(\tau)\to 1$ as $\tau\to\infty$ for any field state, the "memory" of the first detected photon being lost after a long enough time. For coherent states, $g^{(2)}(\tau)=1$. For $g^{(2)}(\tau) < g^{(2)}(0)$, the probability of detecting a second photon after delay time τ decreases and this indicates a bunching of photons. For a thermal field, $g^{(2)}(0)=2$. On the other hand, for $g^{(2)}(\tau) > g^{(2)}(0)$,the probability of detecting a second photon increases with the delay time. This is photon antibunching which, as we have previously discussed, this is a nonclassical effect. We can now explain why.

Let us first consider a single-mode field for which

$$g^{(2)}(\tau) = g^{(2)}(0) = \frac{\langle\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}\rangle}{\langle\hat{a}^\dagger\hat{a}\rangle^2}$$

$$=1+\frac{\langle(\Delta\hat{n})^2\rangle-\langle\hat{n}\rangle}{\langle\hat{n}\rangle^2} \quad \text{……….. (29)}$$

there cannot be any photon antibunching or bunching for a single-mode field as $g^{(2)}(\tau)$ is independent of the delay time τ. We can write

$$g^{(2)}(0) = 1 + \frac{\int p(\alpha)[|\alpha|^2-\langle\hat{a}^\dagger\hat{a}\rangle]^2 d^2\alpha}{\langle\hat{a}^\dagger\hat{a}\rangle^2} \quad \text{…….. (30)}$$

Where $\langle\hat{a}^\dagger\hat{a}\rangle = \int P(\alpha)\,|\alpha|^2 d^2\alpha$ …….. (31)

For a classical field state where P(α)≥0, we must have $g^{(2)}(0)\geq 1$. But for a nonclassical field state it is possible to have $g^{(2)}(0) < 1$, which, as previously stated, may be interpreted as a quantum mechanical violation of the Cauchy inequality. The condition $g^{(2)}(0) < 1$ is the condition that the Q-parameter

$$Q=\frac{\langle(\Delta\hat{n})^2\rangle-\langle\hat{n}\rangle}{\langle\hat{n}\rangle}$$

be negative; that means this is the condition for sub-Poissonian statistics. Indeed Q and $g^{(2)}(0)$, for a single-mode field, are simply related:

$$Q=\langle\hat{n}\rangle[g^{(2)}(0)-1] \quad \text{…….. (32)}$$

The fact that Q < 0 when $g^{(2)}(0) < 1$ has led to some confusion regarding the relationship of sub-Poissonian statistics and photon antibunching.

Again, for a single-mode field, $g^{(2)}(\tau) = g^{(2)}(0)$=constant and thus there can be no photon antibunching (or bunching for that matter). Bunching and antibunching occur only for multimode fields.For such fields in states with P function $P(\{\alpha_i\})$,where $\{\alpha_i\}$denotes collectively the set of complex phase-space variables associated with each of the modes, the modes distinguished by the labels i and j, it can easily be shown that

$g^{(2)}(0)=1+\frac{\int P(\{\alpha_i\})[\sum_j|\alpha_j|^2-\langle\widehat{a_j}^\dagger\widehat{a_j}\rangle]^2 d^2\{\alpha_i\}}{(\sum_j\langle\widehat{a_j}^\dagger\widehat{a_j}\rangle)^2}$ …….. (33)

where $d^2\{\alpha_i\}=d^2\alpha_1 d^2\alpha_2$...

Again, for classical field with P($\{\alpha_i\}$)≥0, $g^{(2)}(0)\geq 1$.

The Cauchy–Schwarz inequality applied to the corresponding classical coherence function $\gamma^{(2)}(\tau)$ implies that for classical fields it should always be the case that $g^{(2)}(\tau)\leq g^{(2)}(0)$, which does not allow for photon antibunching . The condition for antibunching, that $g^{(2)}(\tau) > g^{(2)}(0)$, is therefore an indication of nonclassical light as is the condition $g^{(2)}(0) < 1$. Remembering that $g^{(2)}(\tau)\rightarrow 1$ for τ →∞,it follows that the condition $g^{(2)}(0) < 1$ implies photon antibunching, except (and this is an important exception)in the case of a single-mode field or,if $g^{(2)}(\tau)$ is constant, for some other reason. But the converse is not necessarily true. The condition for antibunching, $g^{(2)}(\tau) > g^{(2)}(0)$ does not imply sub-Poissonian statistics, $g^{(2)}(0) < 1$.

Difference between Photon anti-bunching and Sub-poissionian statistics.

Photon antibunching is characteristic of a light field with photons more equally spaced than a coherent laser field, e.g. light emitted from a single atom. It can also refer to sub-Poisson photon statistics, that is a photon number distribution for which the variance is less than the mean. Photon antibunching and sub-Poisson photon statistics reveal the quantum nature of light and have been studied in many works on quantum optics . Here we consider some mathematical aspects of the theory of such states. We give a rigorous formulation of sub-Poisson statistics and antibunching and prove the impossibility of classical probabilistic representations of quantum correlation functions in these cases. It shows that the classical underlying random fields do not exist in the cases of sub-Poisson statistics and antibunching.

For sub-poissonian statistics:

The sub-Poisson photon statistics is such a photon number distribution for which the variance is less than the mean. Remind that for the Poisson statistics (for coherent states of light) they are equal. We consider a single-mode radiation field. The variance of the photon number distribution is

$\langle\Delta n^2\rangle=\langle n^2\rangle-\langle n\rangle^2$, ………………..(30)

Where n=a^*a …………………(31)

and the commutation relations for the annihilation and creation operators a and a^* is [a, a^*] = 1…………………….(32)

We denote here ⟨n⟩ = ⟨ψ|n|ψ⟩ where the vector ψ is a unite vector in a dense domain of subspace of unite vectors . By using the commutation relations one gets

K ≡$\langle\Delta n^2\rangle-\langle n\rangle = \langle a^{*2}a^2\rangle-\langle a^*a\rangle^2$……….. (33)

Note that one has

K = $\langle a^{*2}a^2\rangle-\langle a^*a\rangle^2 = ||a^2\Psi||^2-||a\Psi||^4$……… (34)

Definition:

If for a unite vector ψ one has K = $||a^2\Psi||^2-||a\Psi||^4 < 0$, ………..(35)

then the vector ψ is called having the sub-Poisson statistics.

For Photon anti-bunching:

Antibunching is the violation of the inequality

P(τ) ≤ P(0), τ ≥ 0…………… (36)

Here

P(τ) = ⟨ψ|E^-(r,t)E^-(r,t + τ)E^+(r,t + τ)E^+(r,t)|ψ⟩,……… (37)

we consider stationary states ψ and do not indicate dependence from the space position of the detector r. The inequality can be understood in the following way. If I(t) is a (classical) stationary random process then, due to the Schwarz inequality, one has:

E(I(t)I(t + τ)) ≤ $(\mathrm{E}I^2(\mathrm{t}))^{1/2}(\mathrm{E}I^2(\mathrm{t}+\tau))^{1/2}$ = EI^2(0)………. (38)

The inequality (38) is the same as (36) if we set

P(τ) = E(I(t)I(t + τ)). …………….(39)

We have proved the following.

Theorem. If for some state ψ and for $\tau > 0$ one has

$P(\tau) > P(0)$, ……………… (40)

then this state does not admit a classical description. In this situation one has antibunching. Such states ψ are called non classical states.

Conclusions:

In this work, we have reviewed various fundamental theoretical aspects related to the quantization of the electromagnetic field, photon number distributions, and non-classical properties of light. Starting from the classical Maxwell equations, we explored the transition to quantum optics through the process of electromagnetic field quantization. This foundational shift allows us to understand light as a quantized field, where photons exhibit statistical properties that differ between classical (thermal) and quantum (coherent) states. The discussion on photon number distributions for thermal and coherent (example:laser) fields draw a difference between classical and quantum light. In a thermal field, photon statistics follow Bose-Einstein distributions is followed by photon in thermal field,but in case of coherent states, (laser light) photon follows Poissonian statistics.

From that we can clearly draw the quantum mechanical nature of coherent light. The study of non-classical light properties, including squeezed states and photon anti-bunching, is essential in understanding the unique behavior of quantum light. Non-classical states, characterized by negative or singular Glauber-Sudarshan P functions, exhibit phenomena that cannot be explained by classical optics. Squeezed states, which reduce noise in one quadrature at the expense of increasing it in another, are of particular importance for applications in high-precision measurements, such as gravitational wave detection. Photon anti-bunching, which is a hallmark of single-photon sources, further demonstrates the particle-like nature of light in quantum systems. Overall, the review study of non-classical light has many applications in quantum information science, quantum cryptography, quantum computing and quantum teleportation. The ability to generate and manipulate non-classical states of light is crucial for advancing technologies based on quantum optics, offering promising directions for future research in both fundamental physics and practical implementations.

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