Anti Fuzzy Line Graph of Complementary Anti Fuzzy Graph

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Abstract
The objective of this paper is to analyze the characteristics of anti fuzzy line graph. Anti fuzzy line graph is computed from some kinds of complementary anti fuzzy graphs. This paper derives and compares the result for strong and self complementary anti fuzzy graphs.
Keywords: Anti Fuzzy Graph (AFG), Anti Fuzzy Line Graph (AFLG), Strong AFG, Self Complementary Anti Fuzzy Graph.

Introdution

In 1965, [1]L.A. Zadeh introduced the fuzzy sets to deal the problem of uncertainty. Fuzzy set is the generalization of crisp set which permits the membership value is from 0 to 1. [2]Rosenfeld (1975) introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. [3]Mordeson(1994) introduced the concept of complement of fuzzy graphs. [4]M.S sunitha and A.Vijayakumar gave a modified definition of complement of fuzzy graph and proved a necessary and sufficient condition for a fuzzy graph to be self complementary. [5] R.Muthuraj and A.Sasireka introduced the concept of an Anti fuzzy graph and its degree. [6] R.Muthuraj and A.Sasireka noted the definition of effective and week edges. [7] A. Nagoorgani and J. Malarvizhi gave the result for size of Fuzzy graph and its complement. [8] A. Nagoorgani and S. Shajitha Begum introduced the degree, order and size of Intuitionistic fuzzy graphs and properties of fuzzy line graph. In which, they have categorized the minimum and maximum effective degree of IFG. This idea is help to develop the effective edges of AFG and its characteristics when the graph is complete. [9] P.Kousalya and Dr.V.Ganesan investigate the properties of self complementary and complete fuzzy graphs with examples.

2. Preliminaries

A fuzzy graph G=(V,E) is a set of functions V : X→ [0,1] and E : X × X→[0,1] such that $\mu(x,y) \leq \mu(x) \wedge \mu(y)$ for all x and y in V. $\wedge$ represents the minimum. Anti Fuzzy Graph $G_A(\sigma, \mu)$ consist of vertices($\sigma: S \to [0,1]$) and edges($\mu: S \times S \to [0,1]$) such that $\mu(x,y) \geq \max(\sigma(x), \sigma(y))$ for all x and y in S.

The degree of a vertex $\sigma(x)$ of an anti fuzzy graph is sum of degree of membership of all those edges which are incident on it. It is denoted by, d_{G_A} $(\sigma(x)) = \sum_{x \neq y} \mu(x,y)$. Anti fuzzy graph $G_A(\sigma,\mu)$ is called strong if μ(x,y) = max($\sigma(x),\sigma(y)$) $\forall$ (x,y) in μ. $G_A(\sigma,\mu)$ is called complete if μ(x,y) = max($\sigma(x),\sigma(y)$) for al x, y in σ. The degree of a vertex $\sigma(u)$ of an anti fuzzy graph is sum of degree of memebership of all those edges which are incident on vertex $\sigma(u)$ and is denoted by d_{G_A} $\sigma(u)$ = d($\sigma(u)$)= $\sum_{u \neq v} \mu(u,v) = \sum_{uv \in E} \mu(u,v)$.

3. Major Part of work

This section exhibit the important kinds of anti fuzzy graph to achieve the main results.

3.1 Strong Anti Fuzzy Graph

A fuzzy graph G_A(σ,μ) is said to be strong anti fuzzy graph.If μ (u, v) = σ (u) ∨ σ (v) or μ (u, v) = max(σ(u), σ(v)) ∀ u,v∈μ*.

Example – 3.1

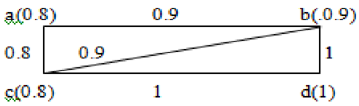

Fig 1: Strong Anti Fuzzy Graph $G_A(\sigma,\mu)$

3.2 Complementary Anti Fuzzy Graph

1. The complement of Anti fuzzy graph $\bar{G}_A(\bar{\sigma},\bar{\mu})$ is derived from an Anti fuzzy graph $G_A(\sigma,\mu)$ where $\sigma = \bar{\sigma}$ and $\bar{\mu}$(x,y) = 0 if μ(x,y) > 0 and $\bar{\mu}$(x,y) = max$(\sigma(x),\sigma(y))$.
2. The modified definition of complement of anti fuzzy graph is denoted by $\bar{G}_A(\bar{\sigma},\bar{\mu})$. Where $\sigma = \bar{\sigma}$ and $\bar{\mu}$(x,y) = μ(x,y) - max$(\sigma(x),\sigma(y))$.

Anti fuzzy is said to be self complementary if there exist an isomorphism between the underlying graph and its complement. That is, G_A(σ,μ) = $\bar{G}_A(\bar{\sigma},\bar{\mu})$.

Example – 3.2

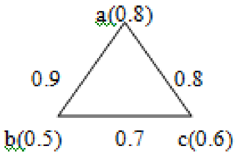

Fig 2(a) : Anti Fuzzy Graph $G_A(\sigma,\mu)$

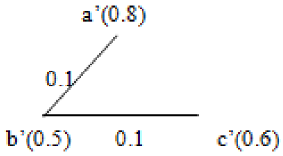

Fig 2(b) : Complement Anti Fuzzy Graph $\bar{G}_A(\bar{\sigma},\bar{\mu})$

3.3. Anti Fuzzy Line Graph

G_A be any anti fuzzy graph. The line graph of G_A, denoted by $L(G_A) = (X,Z)$, is the graph with the set of vertices $X = \{\{e\} \cup \{uv\} / e \in E, u,v \in V, e=uv\}$ and the set of edges $Z = \{S_eS_f / S_e \cap S_f \neq \emptyset, e,f \in E, e \neq f\}$.

Where $S_e = \{\{e\} \cup \{uv\} / e \in E, u,v \in V\}$.

1. Let (σ,μ) be an anti fuzzy sub graph G_A. Define the anti fuzzy subset (λ,ω) of (X,Z) respectively as follows,

   $\forall\, S_e \in Z$, $\lambda(S_e) = \mu(e)$;

   $\forall\, S_eS_f \in W$, $\omega(S_eS_f) = \max\{\mu(e), \mu(f)\}$.

2. (λ,ω) is an anti fuzzy sub graph of $L(G_A)$, called the anti fuzzy line graph corresponding to (σ,μ).

Example – 3.3

Fig 3 (a) : uninodal Anti Fuzzy Graph $G_A(\sigma,\mu)$ 3(b) : $L(G_A(\lambda,\omega))$: Line graph $L^1(G_A)$

4. Main Results

4.1 Strong Anti Fuzzy Line Graphs

This topic investigates the strong anti fuzzy graph and its line graph. The aim of this topic is to claim the isomorphism between strong anti fuzzy graphs and its anti fuzzy line graphs.

Proposition-1: Every AFLG is SAFG.

For every anti fuzzy graph $G_A(\sigma,\mu)$, $\mu(x,y) \geq \max(\sigma(x),\sigma(y))$ for all x and y in S.

By (3.3) The Fuzzy line graph of this graph $L(G_A)$, $\omega(S_eS_f) = \max\{\mu(e), \mu(f)\}$.

Clearly, all the edges are effective in the resultant graph.

That is, Every Anti fuzzy line graph is strong.

Proposition-2: Anti fuzzy graph with 2 vertices corresponding to an isolated anti fuzzy line graph. Since, every vertex of Anti fuzzy line graph which associate with the edges of underlying graph.

Proposition-3: Every strong and complete AFG is corresponding to strong AFLG. But the converse need not be true.

For any anti fuzzy graph $G_A(\sigma,\mu)$, $\mu(x,y) \geq \max(\sigma(x),\sigma(y))$ for all x and y in S.

By (3.3) all the edges of anti fuzzy line graph are effective.

That is, Every Anti fuzzy line graph is strong.

Similarly, For any complete anti fuzzy graph $G_A(\sigma, \mu)$, μ(x,y) = max($\sigma(x), \sigma(y)$) for all x and y in S. Obviously, by the AFLG, all the edges are effective. That is, the complete AFG is corresponding to Strong AFLG. But the converse need not be true.

Example (3.3) shows the relevance of this proposition.

4.2 Self Complement Anti Fuzzy Graphs and its Line graph

Self complementary anti fuzzy graph preserves the isomorphism between the underlying graph and its complement. This section observes the anti fuzzy line graph obtained from self complementary anti fuzzy graph.

Proposition-4: Self Complement AFG exists only when the underlying graph must be strong anti fuzzy graph.

For the underlying graph $G_A(\sigma, \mu)$, μ(x,y) = max($\sigma(x), \sigma(y)$) for all x and y in S.

The complement of the above graph $G_A(\sigma, \mu)$, $\sigma = \bar{\sigma}$ and $\bar{\mu}$(x,y) = μ(x,y) – $\max(\sigma(x), \sigma(y))$.

From this, μ(x,y) + $\bar{\mu}$(x,y) = $\max(\sigma(x), \sigma(y))$. It must strong for both underlying and resultant graph. The following case is the suitable example of this proposition.

Example – 4.1

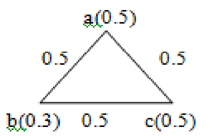

Fig 4(a): Anti Fuzzy Graph

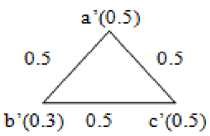

Fig 4(b): Complement of AFG

Proposition-5: Every Strong AFG need not be satisfying Self Complementary.

Example 3.1 shows the proposition. Since, the edge of Complement AFG is strong but not preserves the isomorphism with its underlying graph.

Proposition-6: If a AFG is self complementary then their Anti Fuzzy Line Graph are isomorphic.

If the AFG preserves the Self complement then, $\sigma = \bar{\sigma}$ and $\bar{\mu}$(x,y) = μ(x,y) for all x, y in S.

By Proposition-1 and 3, every strong and complete AFG is corresponding to strong AFLG. Hence, the line graphs of these isomorphic graphs are also isomorphic.

That is, the AFLG of AFG and its complementary graphs are same.

Conclusion

This paper analyzed the anti fuzzy line and it's characteristic. From this observation, this paper concludes that, every kind of anti fuzzy graph preserves the strong anti fuzzy line graph. It also assures the result of strong anti fuzzy line graph need not be corresponding to the strong anti fuzzy graph. In particular, if AFG is self complementary then their Anti Fuzzy Line Graphs are isomorphic. This result may used to investigate more properties about anti fuzzy line graphs.

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