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A STUDY ON CHROMATIC TOTAL DOMINATION IN JAHANGIR GRAPH

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Abstract
Let $G = (V, E)$ be a simple, finite and undirected graph and without isolated vertex. A Subset D of V $(D \subseteq V)$ is said to be a chromatic total dominating set if D is a total dominating set and $\chi(< D >)$ $=\chi(G)$. The minimum cardinality of the chromatic total dominating set is called a chromatic total dominating number $\gamma_{ch}^{t}(G)$.In this paper ,we discuss the chromatic total domination in Jahangir graph.

Keywords: *Jahangir graph, chromatic total domination, chromatic total domination number.*
MSC Subject Classification: *05C69*

Introduction

Let $G = (V, E)$ be a simple, finite and undirected graph and without isolated vertex. A subset D of V is said to be dominating set if for every v in $V - D$ there exist a vertex u in D such that u and v are adjacent. The minimum cardinality of a dominating set of G is called the domination number of G and is denoted by $\gamma(G)$. Cockayne,Dawes & Hedetniemi[2]introduced total domination in graphs.A subset D of V is a total dominating set if $< D >$ has no isolates.The minimum cardinality of a total dominating set of G is called total domination number and denoted by $\gamma_t(G)$.For recent survey of total domination in graphs can be found in [4].Further notation we refer [3].A set D ⊆ V is said to be chromatic total dominating set and $\chi(< D >)$=$\chi(G)$.The minimum cardinality of chromatic total dominating set of G is chromatic total domination number and denoted by $\gamma_{ch}^{t}(G)$[5].In this paper ,we discuss the chromatic total domination number for jahangir graph.

Chromatic Total Domination in Jahangir Graph

Definition 2.1 [3]Jahangir Graph $J_{n,m}$ for $m \geq 3$ is a graph on nm+1 vertices.(i.e) a graph consisting of cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to eachother on C_{nm}

Examples 2.2 *Consider the following graph*

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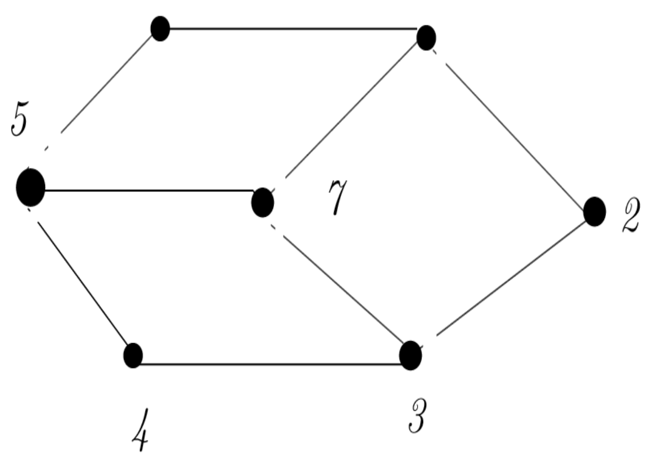

$\chi(G) = 2, D = \{1,7,3\}$ is a total dominating set and $\chi(< D >)$=$\chi(G)$ .Hence ,D is a chromatic total Dominating set of G.

Theorem 2.3 *Let G =$J_{2,m}$ then* $\gamma_{ch}^{t}(G)$=⌈ $\frac{m+1}{2}$ ⌉

Proof. Let $V(J_{2,m})$ =$\{v, v_1, v_2, \dots, v_{2m}\}$ with deg(v)=m.It is clear that $\chi(J_{2,m})$=2

Case (i): m is odd

Let D=$\{v, v_1, v_5, v_9, \dots, v_{2m-1}\}$ be the total dominating set and $\chi(< D >)$=2=$\chi(G)$.Therefore ,D is the chromatic total dominating set .Hence $\gamma_{ch}^{t}(J_{2,m}) \leq \frac{m+1}{2}$.

Sub case(i) :

Suppose if v ∉ D,no vertices of v_i (i=1,2,...,2m) is totally dominated and $\chi(< D > -\{v\}) \neq 2$.

Therefore, D is not a total dominating set of $J_{2,m}$.

Sub case(ii) :

Suppose if v_i ∉ D for some i≠2m-1,no vertex v in D ,suchthat vv_{i-1},vv_{i+1} ∈ $E(J_{2,m})$.Therefore,D is not a total dominating set of $J_{2,m}$.

Sub case (iii) :

Suppose if v_{2m-1} ∉ D,there is no vertex v in D such that $vv_{2m-2} \in E(J_{2,m})$.Therefore,D is not total dominating set of $J_{2,m}$.

The above cases indicated that there is no chromatic total dominating set D_1 such that $|D_1|$ ≺ $|D|$ and D_1 ⊆ D.Therefore $\gamma_{ch}^{t}(G)$=⌈ $\frac{m}{2}$ ⌉+1.

Case (ii):m is even

Let D=$\{v, v_1, v_5, v_9, .. v_{2m-3}\}$ be the total dominating set. $\chi(< D >)$=2=$\chi(G)$. Therefore, D is chromatic total dominating set. $\gamma_{ch}^{t}(J_{2,m}) \geq \frac{m}{2}$+1.

Sub case(i) :

Suppose if v ∉ D,no vertices of v_i (i=1,2,...,2m) is totally dominated and $\chi(< D > -\{v\}) \neq 2$.

Therefore, D is not a total dominating set of $J_{2,m}$.

Sub case(ii) :

Suppose if v_i ∉ D for some i,no vertex v in D ,suchthat vv_{i-1},vv_{i+1} ∈ $E(J_{2,m})$.Therefore,D is not a total dominating set of $J_{2,m}$.

The above cases indicated that there is no chromatic total dominating set D_1 such that $|D_1|$ ≺ $|D|$ and D_1 ⊆ D.Therefore $\gamma_{ch}^{t}(G)$=⌈ $\frac{m}{2}$ ⌉+1.

Theorem 2.4 *Let G =$J_{3,m}$ then* $\gamma_{ch}^{t}(G)$=*m+3*

Proof. Let $V(J_{3,m})$ =$\{v, v_1, v_2, \dots, v_{3m}\}$ with deg(v)=m.Clearly,$J_{3,m}$ contains the cycle of length 3m and has exactly m cycle of length 5($i.eC_5$).Also,$\chi(J_{3,m})$=3.Let $D = \{v, v_1, v_4, \dots, v_{3m-2}\}$ of be the total dominating set $J_{3,m}$.But $\chi(< D >)$=2≠ $\chi(J_{3,m})$.Let D_1=D∪ $\{v_2, v_3\}$.Therefore,$\chi(< D_1 >)$=3,Since ,$< D_1 >$ contains C_5,$vv_1v_2v_3v_4v$).Hence D_1 is the chromatic total dominating set .$\gamma_{ch}^{t}(D_1) \leq |D_1|$=$|D|$+2=m+1+2=m+3.

Case (i):

Suppose if v ∈ D ,no vertices of v_i(i=1,2,...,3m) is totally dominated and $\chi(< D > -\{v\}) \neq 3$.Therefore,D is not a total dominating set of $J_{3,m}$.

Case (ii):

Suppose if v_i ∉ D for some i=1,4,...,3m-2,no vertex v in D ,suchthat vv_{i-1},vv_{i+1} ∈ $E(J_{3,m})$.Therefore,D is not a total dominating set of $J_{3,m}$.

Case (iii):

If $\{v_2, v_3\}$ ∉ D_1 ,then $\chi(< D_1 >)$ ∉ 3.Therefore,D is not a total dominating set of $J_{3,m}$.For $\chi(< D_1 >)$=3 instead of v_2, v_3.we replace the vertices $\{v_{i-1}, v_{i-2}\}$ for i=7,10,...,3m+1.Therefore,D_2=D∪ v_{i-1}, v_{i-2} with $|D_1|$=$|D_2|$.

The above cases indicate that there is no chromatic total dominating set D_2 such that $|D_2|$ < $|D_1|$ and $D_2 \subseteq D_1$.Therefore,$\gamma_{ch}^{t}(J_{3,m})$=m+3.

Theorem 2.5 *Let G=$J_{4,m}$,then* $\gamma_{ch}^{t}(J_{4,m})$ =$\frac{mn}{2}$

Proof. let $V_{J_{4,m}}$=$\{v, v_1, v_2, \dots, v_{4m-1}, v_{4m}\}$ with deg (v)=m.Clearly,$\chi(J_{4,m})$=2.Let D=$\{v_1, v_2, v_5, v_6, \dots, v_{4m-2}, v_{4m-1}\}$ be the total dominating set of $J_{4,m}$,then $\chi(< D >)$=2=$\chi(< 4, m >)$.D is a chromatic total dominating set.Therefore,$\gamma_{ch}^{t}(J_{4,m})$ =$\frac{mn}{2}$.

Theorem 2.6 *Let G=$J_{5,m}$then* $\gamma_{ch}^{t}(G)$=*2m+5*

Proof. Let $V(J_{5,m})$=$\{v, v_1, v_2, \dots, v_{5m}\}$ with deg(v)=m.Clearly,$J_{5,m}$ contains cycle of length 5m and has exactly m cycle of length 7(i.e C_7).Also,$\chi(J_{5,m})$=3.Let $D = \{v, v_3, v_4, v_8, v_9, \dots, v_{5m-2}\}$be the total dominating set of $J_{5,m}$.But $\chi(< D >) = 2 \neq \chi(J_{5,m})$.Let D_1=$D \cup \{v_2, v_5, v_1, v_6\}$.Therefore,$\chi(< D_1 >) =$ 3,since $< D_1 >$contains C_7,($vv_1v_2v_3v_4v_5v_6v$).Hence,D_1 is the chromatic total dominating set .$\gamma_{ch}^{t}(G) \leq |D|$=$|D| +$ 4=2m+1+4=2m+5.

Case (i):

Suppose if v $\not\in$ D,no vertices of $v_i(i = 1,2,\ldots,5m)$is totally dominated and $\chi(< D - v >) \neq 3$.Therefore,D is not a total dominating set of $J_{5,m}$

Case (ii):

Suppose if $v_i \not\in$ D for some i=1,4,...,5m-2 ,no vertex v in D ,$vv_{i-1}, vv_{i+1} \in E(J_{5,m})$.Therefore D is not a total dominating set of $J_{5,m}$.

Case (iii):

If $v_2.v_5, v_1, v_6 \not\in D_1$,then $\chi(< D_1 >) \not\in 3$.Therefore,D is not a total dominating set of $J_{5,m}$.For $\chi(< D_1 >)$=3,instead of $\{v_2, v_5, v_1, v_6\}$ we can replace the vertices $\{v_{i-1}, v_{i-4}, v_{i-5}, v_i\}$ for i=7,10,...,5m+1.Therefore,D_2=D∪ $\{v_{i-1}, v_{i-4}, v_{i-5}, v_i\}$ with $|D_1|$=$|D_2|$.

The above cases indicate there is no chromatic total dominating set D_2 such that $|D_2| < |D_1|$ and $D_2 \subseteq D_1$.Therefore,$\gamma_{ch}^t(J_{5,m})$=2m+5

Observation 2.7 $\gamma_{ch}^t(J_{n,m}) = \begin{cases} (n+2) + (m-1)\gamma_t(p_{n-1}) if n = 5,9,\ldots & n \equiv 1(mod 4) \\ (n+2) + (m-1)\gamma_t(p_{n-3}) if n = 3,7,11,\ldots & n \equiv 3(mod 4) \\ mn if n \equiv 0(mod 4) \\ \frac{mn}{2} + \left\lceil \frac{m}{2} \right\rceil - m + 1 if n \equiv 2(mod 4) \end{cases}$

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