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ANALYSIS OF DIFFERENT APPROACHES USED TO SOLVE TRAVELLING SALESMAN PROBLEM

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Abstract:

The travelling salesperson problem (TSP) is one among the globally recognized and broadly studied problems and is known to be an NP-Hard problem in the field of operational research. It is a mathematical problem where one needs to find the shortest possible route in a collection of cities by passing through each one city exactly once. In order to solve this problem in polynomial time we don't have any well suitable algorithm till date. Even though we have a variety of algorithms so far that provide near optimal solutions. To solve this problem two broad categories of algorithms are used so far one of the categories contains exact algorithms and the other one contains heuristic algorithms. In this paper we are going to review some of these algorithms which are given by different researchers in the past times and considered to be better approaches to solve this problem. To provide a better review we did a thorough analysis of these papers and came out with the brief introduction about the techniques/approaches used and their results.

Keywords: TSP, DP, GA, ACO, PSO.

Introduction:

The Travelling salesperson problem is a familiar NP-hard problem; this infers that there exists no solitary algorithm to unravel it in the polynomial time. The nominal projected time to get optimum solution is exponential [1]. Even though its statement is so easy, yet it remains one of the most bewildering problems in Operational Research. The extensive use of TSP is sharing of assets or items systematically, determining of a shortest track, planning traffic lines etc., and also in the regions that do not need routing [1]. Our sole purpose is to present an outline of various heuristic and exact algorithms so far developed for the Travelling Salesperson Problem.

TSP is a problem to discover the utmost shortest suitable path by the salesman to visit *n* cities with the intention of reaching each and every city just once & lastly comes to the initial position with minimum resources consumption as well as time. It can be well portrayed by a graph '*G*' having '*N*' no. of cities and '*E*' no. of paths between cities.

Let G be a graph shown as G = (N, E) where N is a set of vertices showing cities and E is set of edges showing paths. Let M_{ij} be a cost matrix (or distance matrix) associated with E. M_{ij} can be well-defined in Euclidean Space as follows:

$$M_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

And the path with minimum values is considered as shortest.

While discussing about the heuristic approaches, the first thing used was Genetic algorithm (GA), it is a heuristic algorithm extensively used in optimization problems which is basically inspired from 'evolution' a natural phenomenon. It is grounded on "Survival of the Fittest". GA was firstly introduced in the year 1975 by John Holland. On finding its application in the digital world it attracts a large group of researchers towards it. From the day GA enters in digital world it is used for solving numerous problems related to optimization. In Genetic Algorithm, the sequence of strategies used to find the optimal chromosomes from the population are selection, Crossover and mutation in same order. Additional modifications are proposed with time to expand its capability. It can be enhanced primarily by presenting diverse types of selection operator like rank, steady state, elitism, roulette wheel, tournament, Boltzmann selection etc., crossover operators like one point, k-point, uniform crossover etc. & mutation operator like bit flip, random resetting, inversion, swap, scramble, etc.

M. Dorigo et al. introduced the concept of Ant System (AS) for the first time. Thereafter, it became a provocative issue in this field of research. Numerous new methods are proposed by scholars for its advancement. In ACO, the imitation of real ant behavior is done to get the best suitable path by artificial agents. It is a meta-heuristic approach that is used to solve several optimization problems. Artificial ants are used in place of natural ants to mimic their behavior [2]. The chemical called 'Pheromone' is the basis of working of these ants, which is released by the ants on their track. This pheromone helps other ants to get to the location of food. And the level of pheromone secreted on the track helps other ants to pick the better path. Higher the level of pheromone present, more the number of ants follows that track.

Particle Swarm Optimization (PSO) is also one among the largely recognized heuristic method for optimization. It was primarily proposed by Kennedy & Eberhart in the year 1995. It is based on communal behavior of swarms where the change in path by swarms takes place when they find the best path from ambiguous search space by swapping the information. Here swapping is used to get the information. Due to its easiness as it is simple in implementation & highly convergence behavior, this algorithm has become a popular amongst researchers. It is best suitable for the community of scientific researches and for engineering applications.

In this paper we are going to study the advancements in the above algorithms i.e., what sort of changes are suggested by researchers to improve the working of these algorithms.

Problem formulation:

The formulation of TSP in terms of integer linear programming is as follows:

Allocate number to the cities as 1, 2, 3, …….., n.

$$P_{ij} = \begin{cases} 1 & \text{if path exists between city } i \text{ to city } j \\ 0 & \text{otherwise} \end{cases}$$

For i = 1, ..., n, let υ_i be a model variable and take D_{ij} to be the distance from city i to city j.

Now TSP can be formulated as follows:

Minimize

$$\sum_{i=1}^{n} \sum_{j=1}^{n} Dij\, Pij$$

$$0 \le P_{ij} \le 1$$

$$\upsilon_i \in Z$$

$$\sum_{i \neq j,\, i=1}^{n} Pij = 1$$

$$\sum_{j \neq i,\, j=1}^{n} Pij = 1$$

Where i, j = 1, ... , n;

$$\upsilon_i - \upsilon_j + nP_{ij} \le n - 1 \qquad 2 \le i \ne j \le n.$$

The initial equality set entails that each city should be visited from just one other city, whereas the second equality set entails that from each and every city departure to just one other city is possible.

The final constraints implement that there exists only one single tour that cover all cities i.e. not two or more tours exists that cover all cities.

Applications: Since its formulation, it was studied by a wide variety of researchers in various fields like Mathematics, Computer Science, Physics, Chemistry etc. TSP is broadly and effectively working and found applicable in many areas like in the field of engineering and other streams for architectural designing, telecommunication, routing, planning and various other problems related to industries. Among these applications the most recognizable ones are vehicle routing, DNA sequencing, job sequencing, computer wiring, logistics, resource allocation, microchips manufacturing and many more.

Literature review: In this paper, we going to review various solutions related to TSP. The aim of this literature survey is to analyze and study the available algorithms to reach to an optimal solution for TSP. Since the TSP problem came into reality many researchers started work on it and presented numerous algorithms to solve it. These algorithms are mainly classified as: approximate and exact algorithms [7]. The exact algorithms are said to be those algorithms that solve an optimization problem to give optimal result. The time taken for the running of exact algorithm lies in polynomial factor i.e., O(n!), it is the factorial of number of towns taken in problem, so it is much complex for a problem having 20 – 25 towns only. The exact algorithms used for solving this problem are the dynamic programming, greedy algorithms and branch-and-bound. These algorithms typically entail much time for computation, mainly in case of bigger problems. The algorithms come under this category are the branch-and-bound method proposed by Alison Doig and Alisa Land in 1960s and is employed to solve a wide range of NP- Hard optimization problems, Linear Programming or we can say Linear Optimization is another tool for such optimization problems and Dynamic Programming approach proposed by Richard Bellman in 1950s is also among the major approaches used to solve problems related to optimization. All these exact algorithms discussed so far works rationally fast only in case small sized problem.

Whereas on the other side, the approximate algorithms used to solve this problem are classified as heuristic and meta-heuristic algorithms. The approximate algorithms do not solve an optimization problem to give optimal outcome but they provide the best likely solution which is nearer to the optimal solution. The algorithms come under this category are: Genetic algorithm (GA), Ant Colony Optimization (ACO), Swarm Optimization (SO).

A. Genetic Algorithm

In the year 2001, Kim J. et al. [7] portrayed a paper for solving the TSP with primacy constraints using genetic algorithm. In this paper, the technique used was topological sorting to order the vertices to be traversed by the salesperson. Likewise, they come up with a new crossover operator which is similar in many ways to natural moon i.e., full moon, half-moon etc. are implemented. This operator used for crossover selects an arbitrary subset from population and mix it with the selected parents to produce an offspring. The author compares the afresh developed moon crossover operator with former operators that are OX operator and location-based operator where they found that their performance is nearly equivalent but the OX and location-based operators do not give ideal results for the trials. However, their approach is much effectual for smaller and mid-sized problems but in case of bigger problems it provides best solution without the guarantee of optimality. And the major downside of their paper is that the topological sorting technique they proposed for selecting vertices works only when the graph does not contain any cycle.

In the year 2012, P. Agrawal et al. [8] used a native search technique to upgrade the solution quality. In their paper, they used the technique named Elitism selection for selection which initially imitates the best chromosome to newly generated population and the rest is performed in conventional way. It can speedily escalate the performance of GA because it avoids losing the foremost found solution. In probing procedure, a crossover position on a chromosome is defined and then SCX operator is applied to swap the information. This newly created crossover operator is superior in terms of time & cost as compared to traditional SCX operator. In this mode a proximal optimal solution is attained but not an optimal solution. They presented a relative study among dynamic programming, greedy approach & genetic algorithm for solving TSP. GA appears to be better options for TSP, and nonetheless it is dependent largely on the mode the problem is described & the strategies of mutations & crossover used. They proposed a fresh crossover operator (SCX) which is well in terms of quality of solutions. They used a local search technique to advance the solution quality.

In the year 2013, S. Gupta et al. [9] represented an improved genetic algorithm for TSP problem. In the paper, they used Euclidean formula and form a matrix to compute the distances between various cities visited from the data evaluated. They worked on a symmetric TSP which means that the distance present between two cities is exactly same in both orders while moving from city p to city q and vice versa. They created an initial population at random and then allocate them a fitness value which is taken as the distance between the cities. Subsequently they applied the tournament selection for picking best population from the given set and employed two-point crossover method uniting the facts from heuristic methods & GA for solving the TSP. Lastly interchange mutation is employed for producing new population. It appeared to find better solutions for symmetric TSP but it is not much efficient for asymmetric problems.

In the year 2017, V. Raman et. al. [] presented a paper called Swarmed Genetic Algorithm that blends the power of two algorithms one is genetic algorithm and second one is swarm optimization to overcome the drawbacks of both and uses features of both of these to obtain better outcome. The genetic algorithm present there uses different mutation approaches that are scramble mutation, displacement inversion mutation, interchange mutation so as to get the improved solution. Also, the swarm optimization used there has improved convergence rate, accuracy and flexibility. Lastly, they compared this with existing approaches and find that it provides better result in some cases than the existing ones.

B.Ant Colony Optimization

In the year 2008, B. Li et al. [3] proposed a paper on ACO where the newly coming ants remember the best possible solution present so far. The presented model was labelled as Ants carrying Memory or M_{ant}. In this paper, they did some vital work like introducing the former knowledge of the TSP, Ant System & ACS & outline the parameter used and explain about ants having memory and amalgamate them in ACS, and then lastly the results were obtained by altered ants and compare the effectiveness of each algorithm. This algorithm is able to unite that into at least a proximate optimal solution rapidly. The presented algorithm is so easy to implement as well as better performing. This algorithm suits for smaller and mid-sized problems however in case of bigger problem it may trap into local optima.

In the year 2008, L. Li et al. [4] presented an upgraded ACO for solving TSP. In the paper, they proposed a selection mechanism which is based on Held-Karp lower bound to acquire the optimal path for TSP. It acquires info from gathering of pheromone & heuristic information and uses the H. Karp method for the selection of best possible route.

In the year 2011, Hlaing et. al. [5] gives a solution for TSP based on upgraded Ant Colony Optimization. They come up with an algorithm combined with candidate list approach & dynamic upgrading of heuristic parameter of local search solution. In dynamic candidate list a number of favored nodes are stored in a static list. When an ant travels from one hop to other then it selects the node that is present in the preferred list. This strategy is used to put up searching scheme of ant colony system on bigger data. The upgrading was based on entropy & amalgamate of solution. From their experimental outcomes, the proposed system is much effective in terms of speed (convergence) & the ability of discovering better solutions.

In the year 2012, Hingrajiya et al. [6] proposed a new innovative approach for advancement in ACO. In this paper, they proposed unvarying distribution approach of initial ants so that there is at least a single ant at each node. This has made the solutions search space greater & the chances of getting best results amplified. The heuristic info is used by the ants for the route selection initially. Here, they use a large set of parameters of heuristic to lesser the effect of pheromone so that ants can pick other paths in generating solution. It provokes them to take closer cities which mean that they are expected to opt to travel along small edges. The paper portrays a study for prevention of stagnation behavior & untimely convergence by using unfluctuating distribution of initial ants.

C. **Particle Swarm Optimization**

In the year 2007, X. H. Shi et. al. [10] presented an algorithm that was based on innovative particle swarm optimization for TSP. An irresolute searching approach & an elimination method for crossover are used to upsurge the speed of convergence when related with the prime algorithms for solving TSP by means of swarm intelligence; it has been demonstrated that the problems with bulky size can be solved with offered algorithm. Furthermore, the comprehensive chromosome technique is castoff to additionally extend the algorithm.

In the year 2010, H. Fan [11] presented a hybrid discrete PSO algorithm which augments adaptive disruption factor, heuristic factor and reversion operator, into the approach. In this work, he employed an update mechanism for kinetic equations to expand the effectiveness of particle swarm optimization (PSO). Here, they custom heuristic factor for search operations i.e. to discover a better route and augments reversion mutant for swapping between the pathways and correlates it with the noise factor. This variation increases the effectiveness significantly irrespective of the convergence velocity or correctness.

In the year 2012, X. Yan et. al. [12] proposed a new PS Optimization algorithm which overpowers the weaknesses of GA like early convergence i.e., providing suboptimal solution. They blend the three approaches i.e., GA-PSO-ACO and come up with a 2-stage hybrid swarm intelligent optimization algorithm that offers an improved and much effective solution to TSP problem.

In the year 2017, Huang et. al. [] proposes an improved FOA which is called as elimination based FOA (fruit fly optimization algorithm) that lies in the category of swarm-intelligent optimization algorithms. In the paper, they did three improvements in the existing fruit fly optimization algorithm to overcome the drawbacks like slow convergence rate, falls in local optima and insufficient optimization precision. The improvements made are vision search process in order to improve the convergence rate, elimination mechanism is added to increase the diversity and added a multiplication operator and a reverse operator. Lastly, they made a comparison to show that their proposed work is better in terms of efficiency than the existing algorithms.

In the year 2020, Zhang et. al. [] proposed an enhanced whale optimization algorithm that belongs to the category of swarm intelligent optimization algorithms. In their paper, they introduced an updated version of the formerly present whale optimization algorithm and compare these and showed the enhanced version is better in terms of effectiveness. In the improved version they introduced Gaussian disturbance, adaptive weight and variable neighborhood search strategy to improve the efficiency of the algorithm.

Conclusion and future work:

After analyzing different research papers taken into account here in this paper from the two categories for solving TSP, we can conclude that the heuristic algorithms like GA, ACO, PSO etc. are found to be much more effective than the conventional algorithms like brute-force search, greedy algorithm, branch and bound methods etc. for large collection of cities but for very small number of cities exact algorithms are used and provide best solutions. The heuristic algorithms are found to be effective and efficient for small and mid-sized problems. Moreover, we can say that heuristic algorithms are better than the conventional ones but still these approaches lack in giving the optimal solution for too larger TSP problems. These are suffering with certain issues like stagnation behavior in case of ACO, convergence speed in case of PSO and premature convergence in case of GA due to which they are confined to sub-optimal solutions. From the commencement of these approaches, a lot of alterations are done for refining their performance such as different types of selection, mutation, crossover strategies are presented and adopted from time to time in order to advance GA. Different kinds of updating mechanisms and route selection strategies as well as adding memories in ants are employed for the advancement in ACO as well as in PSO. But still there is a lot of space left for betterment of these algorithms as day-by-day new things are added and found to be effective. We can improve the solution by introducing hybrid approaches that overcomes the problems related with existing solutions.

References

[1] Ivan Brezina Jr., ZuzanaCickova, "Solving the Travelling Salesman Problem using the Ant colony Optimization", Management Information Systems, 2011, Vol. (6), No. (4).

[2] Maredia, A., "History, Analysis, and Implementation of Traveling Salesman Problem (TSP) and Related Problems", Doctoral dissertation, University of Houston, 2010.

[3] Alexander, S., "On the history of combinatorial optimization (till 1960)", Handbooks in Operations Research and

Management Science: Discrete Optimization, 2005.
[4] Dantzig, G., Fulkerson, R., & Johnson, S., "Solution of a large-scale traveling-salesman problem", Journal of the operations research society of America, vol. 2, no. 4, pp. 393-410, 1954.
[5] Karp, R. M, "Reducibility among combinatorial problems", Springer US, pp. 85-103, 1972.
[6] Applegate, D. L., Bixby, R. E., Chvatal, V., & Cook, W. J, "The Traveling Salesman Problem: A Computational Study: A Computational Study", Princeton university press, 2011.
[7] C. M. Dorigo, V. Maniezzo, and A. Colorni, "The ant system: Optimization by a colony of cooperating agents," IEEE Transactions on System, Man, and Cybernetics, Part B, vol.26, pp. 29-41, 1996.
[8] Chiung Moon, Jongsoo Kim, GyunghyunChoi ,YoonhoSeo," An efficient genetic algorithm for the traveling salesman problem with precedence constraints", European Journal of Operational Research 140 (2002) 606–617.
[9] Dwivedi, Taruna Chauhan, Sanu Saxena and Princie Agrawal, "Travelling Salesman Problem using Genetic Algorithm", International Journal of Computer Applications (IJCA), 2012, pp. 25-30.
[10] Gupta S, Panwar P. "Solving Traveling salesman problem using genetic algorithm" International Journal of Advanced Research in Computer Science and Software Engineering (IJARCSSE). 2013; 3(6):376–80.
[11] Vikas Raman, Nasib Singh Gill "A Swarmed GA Algorithm for solving Travelling Salesman Problem", Journal of Theoretical and Applied Information Technology (JATIT), 2017. Vol.95. No.16.
[12] Bifan Li, Lipo Wang & Wu Song "Ant Colony Optimization for the Traveling Salesman Problem Based On Ants with Memory" Fourth International Conference on Natural Computation (2009)
[13] L. Li, S. Ju, Y. Zhang, Improved Ant Colony Optimization for the Traveling Salesman Problem, International Conference on Intelligent Computation Technology and Automation, IEEE, 2008.
[14] Zar Chi Su SuHlaing and May Aye Khine, "Solving Traveling Salesman Problem by Using Improved Ant Colony Optimization Algorithm", International Journal of Information and Education Technology, Vol. 1, No. 5, December 2011.
[15] Hingrajiya KH, Gupta RK, Chandel GS. "An Ant Colony Optimization Algorithm for Solving Traveling Salesman Problem" International Journal of Scientific and Research Publications. 2012; 2(8):1–6.
[16] X.H. Shi , Y.C. Liang, H.P. Lee, C. Lu, Q.X. Wang," Particle swarm optimization-based algorithms for TSP and generalized TSP", Information Processing Letters 103 (2007) 169–176,2007.
[17] Huilian FAN," Discrete Particle Swarm Optimization for TSP based on Neighborhood", Journal of Computational Information Systems 6:10 (2010) 3407-3414.
[18] Xuesong Yan, Can Zhang, WenjingLuo, Wei Li, Wei Chen and Hanmin Liu "Solve Traveling Salesman Problem Using Particle Swarm Optimization Algorithm" International Journal of Computer Science Issues, Vol. 9, Issue 6, No 2, November 2012.
[19] Huang et al. "An improved fruit fly optimization algorithm for solving traveling salesman problem" Front Inform Technol Electron Eng 2017 18(10):1525-1533
[20] Zhang, J.; Hong, L.; Liu, Q. "An Improved Whale Optimization Algorithm for the Traveling Salesman Problem". Symmetry 2021, 13, 48. https://doi.org/10.3390/ sym13010048