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Analytical Solution To Hygrothermal Effect On FG Piezoelectric Quasicrystal Nanoplates

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Abstract:

In this work, a circular plate composed of functionally graded piezoelectric material and a linked hygrothermal field is numerically depict to show the temperature, moisture, displacement, and stress distributions. The axis of the plate is positioned at the center of the connected field. As the radial distance increases, it is reasonable to suppose that the material's properties in the functionally graded piezoelectric material plate change exponentially. The coupled hygrothermal equations, which generate the coupled hygrothermal field throughout the radius of a revolving circular plate, must be solved before the dynamic equilibrium issue can be resolved. After then, the dynamic equilibrium problem may be resolved. The relationship between the circular plate's hygrothermal behaviors and its inner radius, angular speed, functionally graded index, and hygrothermal index is demonstrated numerically in the last section. Future designs of spinning FGPM circular plates in a coupled hygrothermal field may profit from the findings.

Keywords: FGPM, Hygrothermal Field, Circular Plate, Exponential Law

Introduction:

The functionally graded piezoelectric material (FGPM), a sort of nonhomogeneous composite material, is designed to have its composition and properties vary continuously in space on a macroscopic scale [Mian1998]. Researchers whose fields of research include materials and engineering have taken a growing interest in the structural manufacturers of FGPM due to the amazing capabilities these materials have shown in engineering applications [Dai 2013]. Shechtman first identified quasicrystals (QCs) as novel types of solid materials in the early 1980s from the diffraction pattern of quickly cooling Al-Mn alloys. Due to its quasi-periodic structure, QCs have a variety of advantageous characteristics, including high hardness, high wear resistance, low adhesion, and low porosity. QCs are anticipated to be used as sensors in intelligent frameworks, as thin films, covering engine surfaces, thermal barrier coatings, and coating spacecraft wings due to their quasi-periodic atomic arrangement. QCs have drawn a lot of attention in a variety of research domains, including defect issues and mechanical behavior analysis in the layered smart structure, because of their special qualities and possible applications. [Li, yang, gao 2019]. Many studies on different FGPM structural manufacturers can be found in the scholarly literature.

Arani et al. [Arani2012] used the finite element method to conduct their research and generate three-dimensional solutions for closed and open hollow spheres subjected to an internal pressure and uniform temperature field. To provide an analytical solution to the axisymmetric problem of a long, radically polarised FGPM hollow rotating cylinder, Dai et al. [Dai2012a] made use of ordinary integration. This allowed them to accomplish their goal. The dynamic electromagnet elastic response of the FGPM hollow sphere was then established by Dai et al. [Dai2012b] under the influence of coupled multi-fields. Mao and Fu [Mao2010] used the finite difference method to investigate the nonlinear dynamic response and active vibration control of the FGPM plate. In the presence of a supersonic aerodynamic force, Rezaee and Jahangiri [Rezaee2015] studied the nonlinear, chaotic vibrations and stability of a simply supported functionally graded piezoelectric rectangular plate with a bonded piezoelectric layer. The piezoelectric plate had a functionally graded piezoelectric layer. Based on the three-dimensional theory of piezoelectricity, Wang and colleagues [Wang2010] investigated the axisymmetric bending of circular plates with material properties that varied across the thickness. Li et al. [Li2011] used a direct displacement method to examine FGPM circular plate in three dimensions while the plate was subjected to bending and stress. Zhang and Zhong [Zhang2005] reported a three-dimensional solution for the free vibration of FGPM circular plates.

It is not difficult to find multi-physical evaluations of homogeneous composites in the body of published research. For the dynamic problem of magneto-electro-elastic generically laminated beams, Milazzo [Milazzo2013] offered a brand-new one-dimensional model. Because earlier models were two-dimensional, this one was created in response. Multilayered magneto-electro-elastic plates that were subjected to a combination of clamped and free lateral boundary conditions were examined for free vibration by researchers Chen et al. [Chen2014] using a semi-analytical discrete-layer technique. In an orthotropic laminated hollow cylinder that was subjected to thermal shock and a largely uniform magnetic field, Dai and Wang [Dai2005, Dai2006] made analytical conclusions on the transient response of magnetothermostress and perturbation of the magnetic field vector that were produced. Researchers who are interested in the multi-physical analysis of FGMs have been more numerous in recent years. Li et al. [Li2010] addressed the axisymmetric issue of FGM electro elastic cylinders with analytical modelling while making general adjustments to material properties. The issue was resolved using the Freedholm integral equation. The dynamic thermoelastic behaviour of a double-layered cylinder including a FGM layer when subjected to mechanical and thermal loads, respectively, was studied by Dai and Rao [Dai2013] using the Finite Difference Method and the Newmark Method. A simply supported, functionally graded rectangular plate was subjected to time-dependent thermal pressures, and Vel and Batra [Vel2003] used the power series approach to provide an analytical solution for the three-dimensional thermomechanical deformations. This answer related to the three-dimensional thermomechanical deformations that took place. Dai and Dai [Dai2016] measured the displacement and stress fields within a hollow circular FGM disc that was spinning at an angular acceleration and experiencing a changeable temperature field. The disc was subjected to a changing temperature field while this was being done.

Many scholars have also expressed interest in a multi-physical study of the FGPM. The experiment by Jamia et al. [Jamia2016] focused on two collinear mixed-mode limited-permeable cracks that were inserted into an infinite FGPM medium. Electro-mechanical loading sensitivity was built into the crack surfaces. Using a computational Laplace inversion technique, researchers Akbarzadeh et al. [Akbarzadeh2011] examined the thermo piezoelectric response of an FGPM rod following exposure to a moving heat source.

Both the FG effect and the piezoelectric effect are taken into account by FG piezoelectric QC (PQC) materials, which are anticipated to be employed as sensors and actuators to track and regulate the response of structures. Functionally graded PQCs can increase the reliability and dependability of piezoelectric devices in addition to realizing the conversion of mechanical energy into electrical energy. This inspired the presentation of an accurate solution for a FG layered two-dimensional (2D) PQC plate in this study. This inspired the presentation of an analyt solution for a FG layered two-dimensional (2D) PQC plate in this study.

1. *Formulation of Coupled Hygrothermal Field of Functionally Graded Cylinder and its material characteristics:*

Consider an infinitely long cylinder with hygrothermal conditions. The cylinder further increases the impact of heat retention or dissipation. Pressure and material characteristics are thought to remain constant across the cylinder. The coupled hygrothermal model for the functionally graded cylinder is given by,

$$\rho(r)c_p\frac{\partial T}{\partial t}=D_h(r)\nabla^2 T+\rho(r)c_m(\grave{o}h_{LV}+\gamma)\frac{\partial C}{\partial t} \tag{1}$$

$$\rho(r)c_m\frac{\partial C}{\partial t}=D_m(r)\nabla^2 C+D_m(r)\delta\nabla^2 T \tag{2}$$

Where $\nabla^2=\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right)$ is the Laplacian operator, T is the temperature and C is the moisture potential at time t, $D_h(r)$ and $D_m(r)$ is the thermal diffusion coefficient and vapour diffusion coefficient under no absorption condition respectively. The heat and moisture capacity of the medium are c_p and c_m, respectively. The heat of evaporative phase change is denoted by the symbol h_{LV}, $\rho(r)$ is the material density, and γ is the amount of heat released per unit mass of moisture. The thermogradient coefficient is denoted by the symbol δ, while the ratio of the vapour diffusion coefficient to the total moisture diffusion coefficient is denoted by the symbol $\grave{o}$. Assume that the initial moisture and temperature of the material are C_0 and T_0. The power law dependance on the radial coordinate considering 0 as a constant factor of material property is given by,

$$\rho(r)=\rho_0\xi^n, D_h(r)=D_{h0}\xi^n, D_m(r)=D_{m0}\xi^n \tag{3}$$

The coupled diffusion governing Eqs. (1) and (2) additionally include certain source or sink terms. Eq. (1) represents the balance of thermal energy within the cylinder and Eq. (2) represent the balance of moisture within the medium. Because of the transition from liquid to vapour phase as well as to the heat of absorption or desorption, the last term in the governing equation (1), represents the heat sources or heat sink. The term part in this equation shows the moisture source or sink with regard to the temperature gradient. Using Eq. (3) in Eq. (1) and (2), we get

$$\rho_0 c_p\frac{\partial T}{\partial t}=D_{h0}\nabla^2 T+\rho_0 c_m(\grave{o}h_{LV}+\gamma)\frac{\partial C}{\partial t} \tag{4}$$

$$\rho_0 c_m\frac{\partial C}{\partial t}=D_{m0}\nabla^2 C+D_{m0}\delta\nabla^2 T \tag{5}$$

Define dimensionless variables as,

$$\theta = \frac{T - T_0}{T_0}, \psi = \frac{C - C_0}{\lambda T_0}, \qquad (6)$$
$$\xi = \frac{r}{r_0}, \zeta = \frac{z}{r_0}, t^* = \frac{Lt}{r_0^{\ 2}}$$

Using Eq. (6) in Eq. (4)- (5), after simplification we obtained the hygorthermoelastic model in dimensionless form as,

$$\mathrm{D}\nabla^{2*}\theta = \frac{\partial \theta}{\partial t^*} - \eta \frac{\partial \psi}{\partial t^*} \qquad (7)$$

$$D\nabla^{2*}\psi = \frac{\partial \psi}{\partial t^*} - \lambda \frac{\partial \theta}{\partial t^*} \qquad (8)$$

Where,

$$\nabla^{2*} = \left(\frac{\partial^2}{\partial \xi^2} + \frac{1}{\xi}\frac{\partial}{\partial \xi} + \frac{\partial^2}{\partial \zeta^2} \right), \mathrm{D} = \frac{D_{h0}}{\rho_0 c_p L}, \eta = \frac{c_m \lambda_c (\partial h_{LV} + \gamma)}{c_p} \qquad (9)$$
$$D = \frac{D_{m0} D_{h0}}{\rho_0 c_m L[D_{h0} + D_{m0}\delta(\partial h_{LV} + \gamma)]}, \lambda = \frac{D_{m0}\delta c_p}{c_m \lambda_c [D_{h0} + D_{m0}\delta(\partial h_{LV} + \gamma)]}$$

The thermal and moisture boundary conditions in dimensionless form without any heat source can be expressed as follows:

$$\theta(a,\zeta,t) + \frac{\partial \theta(a,\zeta,t)}{\partial \xi} = 0 \qquad (10)$$

$$\theta(b,\zeta,t) + \frac{\partial \theta(b,\zeta,t)}{\partial \xi} = 0 \qquad (11)$$

$$\psi(a,\zeta,t) + \frac{\partial \psi(a,\zeta,t)}{\partial \xi} = 0 \qquad (12)$$

$$\psi(b,\zeta,t) + \frac{\partial \psi(b,\zeta,t)}{\partial \xi} = 0 \qquad (13)$$

$$\theta(\xi,0,t) = \theta_1 \delta(t) \qquad \theta(\xi,h,t) = \theta_2 \delta(t) \qquad (14)$$

$$\psi(\xi,0,t) = \psi_1 \delta(t) \qquad \psi(\xi,h,t) = \psi_2 \delta(t) \qquad (15)$$

The initial condition for temperature and moisture is given by,

$$\theta(\xi,\zeta,0) = 0 \text{ and } \psi(\xi,\zeta,0) = 0 \qquad (16)$$

From this part we neglect (*) sign for the inconvenience.

2. Solution of the Coupled Hygrothermoelastic Model:

Let us define the integral transform given by E. Marchi and G. Zgrablich [1964] as

$$\overline{f}(n) = \int_a^b \xi f(\xi) S_p(k_1, k_2, \mu_n \xi) \mathrm{d}\xi \qquad (17)$$

And its inversion is given by,

$$f(\xi) = \sum_n \frac{\overline{f_p}(n)}{c_n} S_p(k_1, k_2, \mu_n \xi) \qquad (18)$$

Where
$$c_n = \int_a^b \xi \left[S_p\left(k_1, k_2, \mu_n \xi\right) \right]^2 \mathrm{d}\xi \tag{19}$$

$$\begin{aligned} S_p\left(k_1, k_2, \mu_n \xi\right) = J_p\left(\mu_n \xi\right)\left[G_p\left(k_1, \mu_n a\right) + G_p\left(k_2, \mu_n b\right)\right] - \\ G_p\left(\mu_n \xi\right)\left[J_p\left(k_1, \mu_n a\right) + J_p\left(k_2, \mu_n b\right)\right] \end{aligned} \tag{20}$$

and $J_p\left(\mu_n \xi\right) \& G_p\left(\mu_n \xi\right)$ are Bessel Functions of first and second kind respectively of order p.

Also, μ_n is the positive roots of equation

$$J_p\left(k_1, \mu_n a\right) G_p\left(k_2, \mu_n b\right) - J_p\left(k_2, \mu_n b\right) G_p\left(k_1, \mu_n a\right) = 0 \tag{21}$$

The operational property for the Marchi-Zgrablich integral transform defined in Eq. (17) is given by,

$$\begin{aligned} \int_a^b \xi \left[\frac{\partial^2 f}{\partial \xi^2} + \frac{1}{\xi}\frac{\partial f}{\partial \xi} - \frac{p^2}{\xi^2} f \right] S_p(k_1, k_2, \mu_n \xi) \mathrm{d}\xi = \frac{b}{k_2} S_p(k_1, k_2, \mu_n b) \left[f + k_2 \frac{\partial f}{\partial \xi} \right]_{\xi=b} \\ - \frac{a}{k_1} S_p(k_1, k_2, \mu_n a) \left[f + k_1 \frac{\partial f}{\partial \xi} \right]_{\xi=a} - \mu_n^2 \overline{f}_p(k_1, k_2, \mu_n a) \end{aligned} \tag{22}$$

Applying the transform defined in Eq. (17) to Eqs. (7), (8) and Eqs. (14)-(16) using boundary conditions given in Eqs. (10)-(13) in dimensionless form and operational property in Eq. (22) at $p = 0$, we get

$$\mathrm{D}\left[-\mu_n^2 \overline{\theta}(n, \zeta, t) + \frac{\partial^2 \overline{\theta}(n, \zeta, t)}{\partial \zeta^2} \right] = \frac{\partial \overline{\theta}(n, \zeta, t)}{\partial t} - \eta \frac{\partial \overline{\psi}(n, \zeta, t)}{\partial t} \tag{23}$$

$$D\left[-\mu_n^2 \overline{\psi}(n, \zeta, t) + \frac{\partial^2 \overline{\psi}(n, \zeta, t)}{\partial \zeta^2} \right] = \frac{\partial \overline{\psi}(n, \zeta, t)}{\partial t} - \lambda \frac{\partial \overline{\theta}(n, \zeta, t)}{\partial t} \tag{24}$$

$$\overline{\theta}\left(n, 0, t\right) = \theta_1 \, \delta(t) \qquad \overline{\theta}\left(n, h, t\right) = \theta_2 \, \delta(t) \tag{25}$$

$$\overline{\psi}\left(n, 0, t\right) = \psi_1 \, \delta(t) \qquad \overline{\psi}\left(n, h, t\right) = \psi_2 \, \delta(t) \tag{26}$$

$$\overline{\theta}(n, \zeta, 0) = 0 \text{ and } \overline{\psi}(n, \zeta, 0) = 0 \tag{27}$$

A finite Fourier sine transform [Debnath, Bhatta 2006] is defined by

$$\dot{g}(n, m, t) = \int_0^h g(n, \zeta, t) \sin\left(\frac{m\pi\zeta}{h} \right) \mathrm{d}\zeta \tag{28}$$

with operational property given by,

$$\begin{aligned} \int_0^h \frac{\partial^2 g(n, \zeta, t)}{\partial \zeta^2} \sin\left(\frac{m\pi\zeta}{h} \right) \mathrm{d}z = \frac{m\pi}{h} [(-1)^{m+1} g(n, h, t) + g(n, 0, t)] \\ - \frac{m^2 \pi^2}{h^2} \dot{g}(n, m, t) \end{aligned} \tag{29}$$

Inverse Fourier sine transform of Eq. (29) defined as,

$$g(n, \zeta, t) = \frac{2}{h} \sum_{m=1}^{\infty} \dot{g}(n, m, t) \sin\left(\frac{m\pi\zeta}{h} \right) \tag{30}$$

Applying finite Fourier sine transform defined in Eq. (28) to Eqs. (23), (24) and (27) with boundary conditions given in Eqs. (25)-(26) and using operational property defined in Eq. (29), we get

$$\frac{\partial \dot{\bar{\theta}}(n,z,t)}{\partial \bar{t}} - \eta \frac{\partial \dot{\bar{\psi}}(n,z,t)}{\partial \bar{t}} + \mathrm{D}\left(\frac{m^2\pi^2}{h^2} + \mu_n{}^2\right)\dot{\bar{\theta}}(n,z,t) = \frac{m\pi}{h}\mathrm{D}\,\delta(t)[(-1)^{m+1}\theta_2 + \theta_1] \tag{31}$$

$$\frac{\partial \dot{\bar{\psi}}(n,m,t)}{\partial t} - \lambda \frac{\partial \dot{\bar{\theta}}(n,m,t)}{\partial t} + D\left(\frac{m^2\pi^2}{h^2} + \mu_n{}^2\right)\dot{\bar{\psi}}(n,m,t) = \frac{m\pi}{h} D\delta(t)[(-1)^{m+1}\psi_2 + \psi_1] \tag{32}$$

$$\dot{\bar{\theta}}(n,m,0) = 0 \text{ and } \dot{\bar{\psi}}(n,m,0) = 0 \tag{33}$$

Now define the Laplace transform of a function $h(t)$ as

$$\hat{h}(s) = L[h(t)] = \int_0^\infty h(t)e^{-st}dt,\ s > 0 \tag{34}$$

where s is the Laplace parameter [Debnath, Bhatta 2006].

Applying laplace transform as defined in Eq. (34) to Eq. (31)-(32) applying condition in Eq. (33) and after simplification one can obtain

$$\hat{\dot{\bar{\theta}}}(n,m,s) = -\frac{hm\pi\{\mathrm{D}\lambda[\theta_1 + (-1)^{1+m}\theta_2] + D\eta[-\psi_1 + (-1)^m\psi_2]\}}{Dm^2\pi^2\eta + h^2 s(\eta-\lambda) - \mathrm{D}m^2\pi^2\lambda + h^2(D\eta - \mathrm{D}\lambda)\mu_n^2} \tag{35}$$

$$\hat{\dot{\bar{\psi}}}(n,m,s) = -\frac{m\pi\left\{-\mathrm{D}[\theta_1 + (-1)^{1+m}\theta_2]\left(\frac{Dm^2\pi^2}{h^2} + s + D\mu_n^2\right) + D\left(\frac{\mathrm{D}m^2\pi^2}{h^2} + s + \mathrm{D}\mu_n^2\right)\left(\psi_1 + (-1)^{1+m}\psi_2\right)\right\}}{h\left[-s\eta\left(\frac{Dm^2\pi^2}{h^2} + s + D\mu_n^2\right) + s\lambda\left(\frac{\mathrm{D}m^2\pi^2}{h^2} + s + \mathrm{D}\mu_n^2\right)\right]} \tag{36}$$

Eq. (35) and (36) represent the temperature and moisture distribution in Laplace domain respectively.

Taking inverse Laplace transform of Eq. (35) and (36) we get,

$$\dot{\bar{\theta}}(n,m,t) = -\frac{m\pi\{\mathrm{D}\lambda[\theta_1 + (-1)^{1+m}\theta_2] + D\eta[-\psi_1 + (-1)^m\psi_2]\}}{h(\eta-\lambda)} e^{-\left[\frac{(D\eta - \mathrm{D}\lambda)(m^2\pi^2 + h^2\mu_n^2)}{h^2(\eta-\lambda)}\right]t} \tag{37}$$

$$\dot{\bar{\psi}}(n,m,t) = \frac{m\pi}{h(D\eta - \mathrm{D}\lambda)}\left\{-D\mathrm{D}[(\theta_1-\psi_1) + (-1)^{1+m}(\theta_2-\psi_2)] + \frac{(D-\mathrm{D})[-\mathrm{D}\lambda\theta_1 + D\eta\psi_1 + (-1)^m(\mathrm{D}\lambda\theta_2 - D\eta\psi_2)]}{(\eta-\lambda)} e^{-\frac{(D\eta-\mathrm{D}\lambda)(m^2\pi^2+h^2\mu_n^2)}{h^2(\eta-\lambda)}t}\right\} \tag{38}$$

Taking inverse fourier transform of Eqs. (37)-(38) using Eq. (30) we obtain

$$\bar{\theta}(n,\zeta,t) = \frac{2}{h}\sum_{m=1}^{\infty}\left\langle -\frac{m\pi\{\mathrm{D}\lambda[\theta_1 + (-1)^{1+m}\theta_2] + D\eta[-\psi_1 + (-1)^m\psi_2]\}}{h(\eta-\lambda)} e^{-\left[\frac{(D\eta-\mathrm{D}\lambda)(m^2\pi^2+h^2\mu_n^2)}{h^2(\eta-\lambda)}\right]t}\right\rangle \sin\left(\frac{m\pi\zeta}{h}\right) \tag{39}$$

$$\bar{\psi}(n,\zeta,t) = \frac{2}{h^2}\sum_{m=1}^{\infty}\left\langle \frac{m\pi}{(D\eta-\mathrm{D}\lambda)}\left\{-D\mathrm{D}[(\theta_1-\psi_1) + (-1)^{1+m}(\theta_2-\psi_2)] + \frac{(D-\mathrm{D})[-\mathrm{D}\lambda\theta_1 + D\eta\psi_1 + (-1)^m(\mathrm{D}\lambda\theta_2 - D\eta\psi_2)]}{(\eta-\lambda)} e^{-\frac{(D\eta-\mathrm{D}\lambda)(m^2\pi^2+h^2\mu_n^2)}{h^2(\eta-\lambda)}t}\right\}\right\rangle \sin\left(\frac{m\pi\zeta}{h}\right) \tag{40}$$

Now apply inversion formula defined in Eq. (18) to Eqs. (39) and (40), we get the temperature and moisture distribution as follows:

$$\theta(r,z,t) = \frac{2}{h^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{c_n}\left\langle -\frac{m\pi\{\mathrm{D}\lambda[\theta_1 + (-1)^{1+m}\theta_2] + D\eta[-\psi_1 + (-1)^m\psi_2]\}}{(\eta-\lambda)} e^{-\left[\frac{(D\eta-\mathrm{D}\lambda)(m^2\pi^2+h^2\mu_n^2)}{h^2(\eta-\lambda)}\right]t}\right\rangle \sin\left(\frac{m\pi\zeta}{h}\right) S_0(k_1,k_2,\mu_n\xi) \tag{41}$$

$$\psi(r,z,t)=\frac{2}{h^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{c_n}\left\langle\frac{m\pi}{(D\eta-\mathrm{D}\lambda)}\left\{-D\mathrm{D}[(\theta_1-\psi_1)+(-1)^{1+m}(\theta_2-\psi_2)]+\frac{(D-\mathrm{D})[-\mathrm{D}\lambda\theta_1+D\eta\psi_1+(-1)^m(\mathrm{D}\lambda\theta_2-D\eta\psi_2)]}{(\eta-\lambda)}e^{-\frac{(D\eta-\mathrm{D}\lambda)(m^2\pi^2+h^2\mu_n^2)}{h^2(\eta-\lambda)}t}\right\}\right\rangle\sin\left(\frac{m\pi\zeta}{h}\right)S_0(k_1,k_2,\mu_n\xi) \tag{42}$$

Formulation of Hygrothermoelastic Large Deflection of Cylinder:

We generalized the Basuli's equation of equilibrium for the transient hygrothermal distribution as follows in order to derive the large deflection equation of a heated cylinder based on Berger's approximations [S. Basuli, 1968 and Tara Dhakate, Vinod Varghese & Lalsingh Khalsa (2018)] as:

$$\nabla_1^2(\nabla_1^2-\beta_1^2)w=-\frac{\nabla_1^2 M_g}{D(1-\nu)} \tag{43}$$

Where $\nabla_1^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}$ is Laplacian operator, ω is the normal transverse deflection along the z-direction, ν denotes Poisson's ratio of the cylinder, $D=Eh^3/12(1-\nu^2)$ is the flexural stiffness of the cylinder, ∇_1^2 indicates Laplacian operator and β_1^2 is normalizing constant of integration to be determined from

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2=\frac{\beta_1^2h^2}{12}+(1+\nu)N_g \tag{44}$$

M_g is the hygrothermally induced resultant moment represented as

$$M_g=E\int_0^h z[\alpha_1(T-T_0)+\alpha_2(C-C_0)]dz \tag{45}$$

with α and E are the coefficient of linear thermal expansion and Young's Modulus of the material of the cylinder respectively.

The result of the above heat conduction gives thermally induced resultant force defined as

$$N_g=E\int_0^h[\alpha_1(T-T_0)+\alpha_2(C-C_0)]dz \tag{46}$$

Equations (45) and (46) have to be solved for heated thin clamped supported annular cylinder along the edges for which the boundary conditions are

$$w\big|_{r=a}=0,\ \frac{\partial w}{\partial r}\bigg|_{r=a}=0 \tag{47}$$

Substitute Eqs. (41) and (42) in Eqs. (45) and (46) we obtain hygrothermal moments and resultant forces respectively as,

$$M_T=\frac{2ET_0r_0}{\pi^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{m^2c_n}\{-D\mathrm{D}\lambda\alpha_2(\eta-\lambda)(\theta_3-\psi_3)+[\alpha_1(\mathrm{D}\lambda-D\eta)-\lambda\alpha_2(D-\mathrm{D})]\times$$
$$(\mathrm{D}\lambda\theta_3-D\eta\psi_3)\exp\left[-\frac{(D\eta-\mathrm{D}\lambda)(m^2\pi^2+h^2\mu_n^2)}{h^2(\eta-\lambda)}t\right]\} \tag{48}$$
$$\left[-m\pi\cos\left(\frac{m\pi}{r_0}\right)+\sin\left(\frac{m\pi}{r_0}\right)r_0\right]S_0(k_1,k_2,\mu_n\xi)$$

$$N_T = \frac{2ET_0 r_0}{\pi h}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{mc_n}\{-D\mathrm{D}\lambda\alpha_2(\eta-\lambda)(\theta_3-\psi_3)+[\alpha_1(\mathrm{D}\lambda-D\eta)-\lambda\alpha_2(D-\mathrm{D})] \\ (\mathrm{D}\lambda\theta_3-D\eta\psi_3)\exp\left[-\frac{(D\eta-\mathrm{D}\lambda)(m^2\pi^2+h^2\mu_n^2)}{h^2(\eta-\lambda)}t\right]\}\left(1-\cos\left[\frac{m\pi}{r_0}\right]\right)S_0(k_1,k_2,\mu_n\xi) \tag{49}$$

Where $\theta_3=\theta_1+(-1)^{m+1}\theta_2$ and $\psi_3=\psi_1+(-1)^{m+1}\psi_2$

Putting Eq. (48) in generalized Basuli's equation of equilibrium given by Eq. (43), we get

$$\nabla_1^{\ 2}(\nabla_1^{\ 2}-\beta_1^2)w=F(\xi) \tag{50}$$

Where,

$$F(\xi)=\frac{2ET_0r_0}{\pi^2D(\nu-1)}\nabla_1^{\ 2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{m^2c_n}\{-D\mathrm{D}\lambda\alpha_2(\eta-\lambda)(\theta_3-\psi_3)+ \\ [\alpha_1(\mathrm{D}\lambda-D\eta)-\lambda\alpha_2(D-\mathrm{D})](\mathrm{D}\lambda\theta_3-D\eta\psi_3)\ \exp\left[-\frac{(D\eta-\mathrm{D}\lambda)(m^2\pi^2+h^2\mu_n^2)}{h^2(\eta-\lambda)}t\right]\} \\ \left[-m\pi\cos\left(\frac{m\pi}{r_0}\right)+\sin\left(\frac{m\pi}{r_0}\right)r_0\right]S_0(k_1,k_2,\mu_n\xi) \tag{51}$$

Solution to the above Eq. (50) is given by,

$$w=\frac{2}{a^2}\sum_{n=1}^{\infty}\frac{[J_0(\mu_n\xi)-J_0(\mu_na)]}{\mu_n^{\ 2}(\beta_1^2+\mu_n^{\ 2})[J_0(\mu_na)]^2}\int_a^b\xi J_0(\mu_n\xi)F(\xi)d\xi \tag{52}$$

Where a_i being the root of the equation [Singh, Prasad, Sidhiqui, 1981].

Simplifying Eq. (52) we get

$$w=\frac{2ET_0r_0}{\pi^2a^2D(\nu-1)}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{[J_0(\mu_n\xi)-J_0(\mu_na)][J_0(a\mu_n)+J_0(b\mu_n)]}{m^2c_n\mu_n(\beta_1^2+\mu_n^{\ 2})[J_0(\mu_na)]^2}\left[-m\pi\cos\left(\frac{m\pi}{r_0}\right)+\sin\left(\frac{m\pi}{r_0}\right)r_0\right] \\ \{-D\mathrm{D}\lambda\alpha_2(\eta-\lambda)(\theta_3-\psi_3)+[\alpha_1(\mathrm{D}\lambda-D\eta)-\lambda\alpha_2(D-\mathrm{D})]\times\ (\mathrm{D}\lambda\theta_3-D\eta\psi_3) \\ \exp\left[-\frac{(D\eta-\mathrm{D}\lambda)(m^2\pi^2+h^2\mu_n^2)}{h^2(\eta-\lambda)}t\right]\}\langle aJ_1(a\mu_n)K_0(a\mu_n)-bJ_1(b\mu_n)K_0(b\mu_n)-aJ_0(a\mu_n)k_1(a\mu_n)+bJ_0(b\mu_n)K_1(b\mu_n) \\ +\{a^2[J_0(a\mu_n)^2+J_1(a\mu_n)^2]-b^2[J_0(b\mu_n)^2+J_1(b\mu_n)^2]\}[K_0(a\mu_n)+K_0(b\mu_n)]\mu_n\rangle \tag{53}$$

Eq. (53) represent hygrothermal large deflection of cylinder at an arbitrary point of the functionally graded cylinder. And Using Eq. (44) and (53) we can obtain displacement component.

Conclusion:

We describe the hygrothermal large deflection study of a hollow nano-cylinder made of functionally graded material (FGM) under a coupled hygrothermal field. The research findings might be characterized by the following conclusions:

1. The FGM hollow nano-cylinder's temperature, moisture, displacement, and deflection distributions are affected by the time along the radius under the hygrothermal field.
2. Both the inner and outer surfaces of the FG hollow cylinder are deflection-free.
3. The analytical process described here is widely applicable in contrast to the method proposed by other researchers.

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